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(S,S) INVENTORY POLICIES IN A NONSTATIONARY DEMAND ENVIRONMENT.--ETC(U)  
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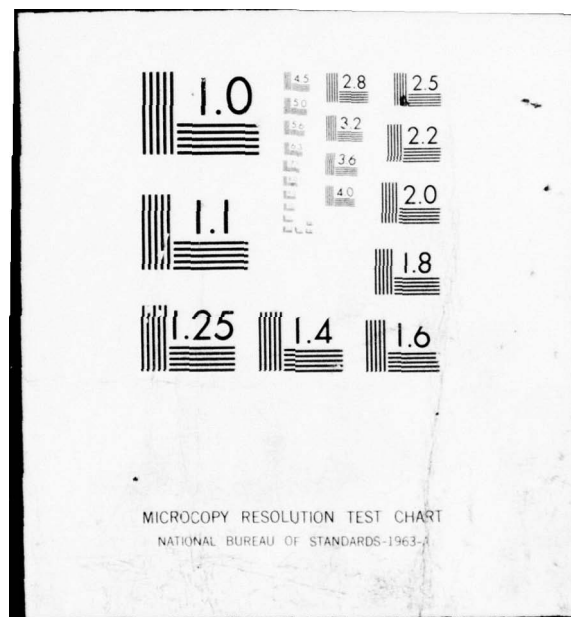
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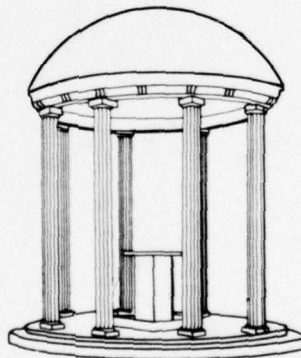
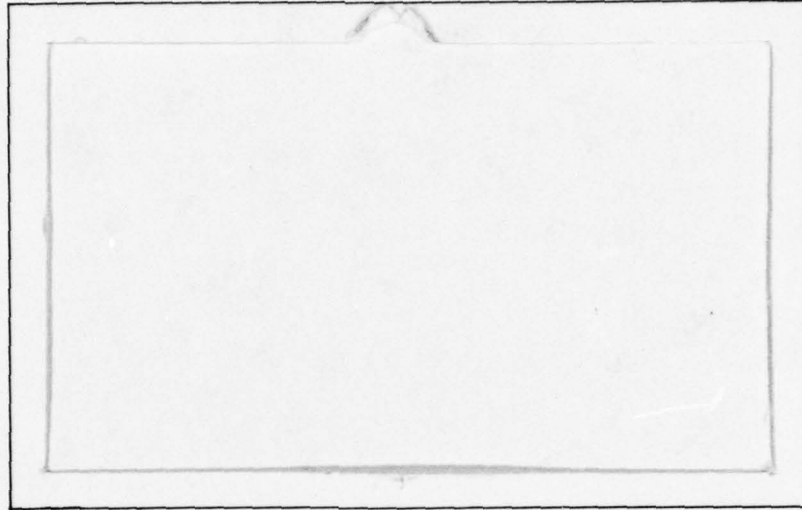
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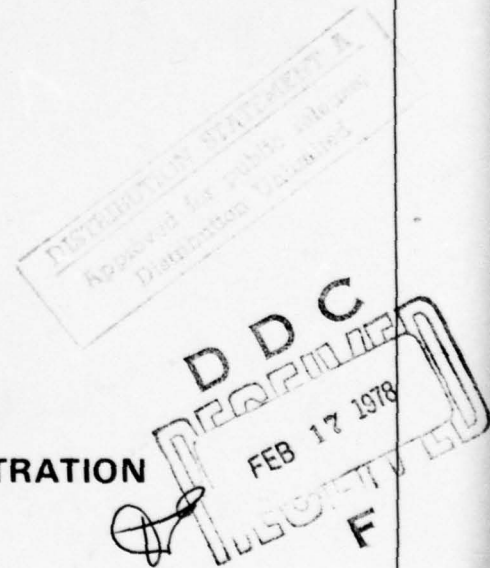
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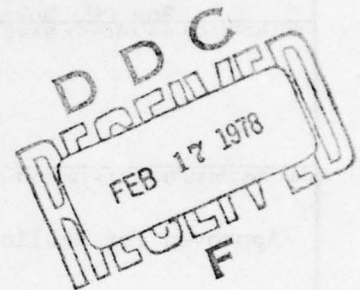


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**(s,s) INVENTORY POLICIES IN A  
NONSTATIONARY DEMAND ENVIRONMENT**

**Technical Report #11  
Appendices  
Ronald Kaufman \***

**April 1977**



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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Scarf (1960) proves the optimality of (s,S) policies for a class of discrete review nonstationary inventory models. A considerable amount of inventory literature concerns computation of optimal and approximately optimal (s,S) policies under the Scarf hypotheses. Little research has dealt, however, with the case of nonstationary demands. This investigation examines the situation in which demand distributions are independent, but not identically distributed, and vary in a cyclic manner. Products that experience seasonal demands are a typical example of such a demand process.		

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A detailed analysis of the nature of optimal policies in the non-stationary environment is presented. The behavior of selected operating characteristics such as period-end inventory, backlogged demand, frequency of stockout, replenishment frequency, and associated costs also are examined. The above performance measures for an individual inventoried good are aggregated over all goods to provide an analysis of the multi-item system behavior.

Approximately optimal (s,S) policies are derived for the nonstationary environment. The policies are based on the power approximation of Ehrhardt (1976), and require knowledge of only the mean and variance of demand. The operating characteristics of approximately optimal policies are compared with those of optimal policies.

The approximately optimal policy rule is examined in a statistical environment that generalizes MacCormick (1974). Policy parameters are revised periodically using a limited history of past demands to estimate the mean and variance of demand. Each time the policy parameters are revised, forecasts of system operating characteristics are calculated from a retrospective simulation employing the same sample of demands that was used to revise the policy.

The statistical phenomena are studied by means of a computer simulation program using time series analysis techniques.

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## Appendix A

### Single-item Data

#### Stationary Model

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

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Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

TABLE A1 AVERAGE TOTAL COST

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, STATIONARY MODEL

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL DP ARE % EXCESS OVER DP VALUE

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	DP	PA	(24,24)
2	8	u	32	25.6	0.1	3.5
4	8	u	32	28.3	0.4	5.5
2	16	u	32	35.9	0.1	2.8
4	16	u	32	39.7	0.1	4.7
2	8	99	32	42.1	0.1	9.2
4	8	99	32	48.0	0.1	12.7
2	16	99	32	57.8	0.3	8.5
4	16	99	32	66.1	0.2	11.5
2	8	u	64	32.8	0.2	2.4
4	8	u	64	35.2	0.3	4.2
2	16	u	64	46.3	0.2	2.2
4	16	u	64	49.6	0.2	4.0
2	8	99	64	50.0	0.1	6.9
4	8	99	64	55.8	0.0	9.7
2	16	99	64	69.3	0.0	6.5
4	16	99	64	77.2	0.1	9.4

TABLE A3 PERIOD-END INVENTORY

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, STATIONARY MODEL

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C(OUT)	C(FIX)	DP	PA	(24,24)
		/C(IN)	/C(IN)			
2	8	4	32	11.5	11.1	11.5
4	8	4	32	13.8	12.4	13.9
2	16	4	32	15.9	16.2	16.9
4	16	4	32	18.8	18.3	20.1
2	8	99	32	28.2	30.1	30.1
4	8	99	32	33.7	34.7	36.7
2	16	99	32	37.9	40.5	41.7
4	16	99	32	46.0	48.0	50.4
2	8	4	64	13.8	12.9	13.4
4	8	4	64	16.0	14.3	15.5
2	16	4	64	19.7	18.3	19.6
4	16	4	64	22.1	20.7	22.5
2	8	99	64	31.8	31.3	33.0
4	8	99	64	36.3	37.3	38.9
2	16	99	64	43.1	44.1	45.3
4	16	99	64	50.6	51.1	53.3



TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, STATIONARY MODEL

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C(OUT)	C(FIX)	DP	PA	(24,24)
		/C(IN)	/C(IN)			
2	8	4	32	0.1716	0.1775	0.1826
4	8	4	32	0.2022	0.2225	0.2244
2	16	4	32	0.1224	0.1129	0.1160
4	16	4	32	0.1482	0.1454	0.1452
2	8	99	32	0.0060	0.0058	0.0086
4	8	99	32	0.0069	0.0058	0.0110
2	16	99	32	0.0041	0.0032	0.0053
4	16	99	32	0.0046	0.0038	0.0071
2	8	4	64	0.1944	0.2047	0.2124
4	8	4	64	0.2184	0.2559	0.2512
2	16	4	64	0.1342	0.1417	0.1347
4	16	4	64	0.1577	0.1661	0.1653
2	8	99	64	0.0056	0.0067	0.0090
4	8	99	64	0.0075	0.0072	0.0117
2	16	99	64	0.0041	0.0040	0.0060
4	16	99	64	0.0046	0.0049	0.0081



TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, STATIONARY MODEL

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	DP	PA	(24, 24)
2	8	4	32	0.1859	0.1922	0.1929
4	8	4	32	0.1842	0.2023	0.1960
2	16	4	32	0.1951	0.1839	0.1838
4	16	4	32	0.1967	0.1962	0.1888
2	8	99	32	0.0093	0.0090	0.0123
4	8	99	32	0.0092	0.0078	0.0132
2	16	99	32	0.0100	0.0079	0.0119
4	16	99	32	0.0096	0.0081	0.0131
2	8	4	64	0.1938	0.2040	0.2058
4	8	4	64	0.1863	0.2131	0.2039
2	16	4	64	0.1955	0.2064	0.1950
4	16	4	64	0.1951	0.2055	0.1975
2	8	99	64	0.0086	0.0101	0.0127
4	8	99	64	0.0098	0.0093	0.0138
2	16	99	64	0.0098	0.0095	0.0130
4	16	99	64	0.0094	0.0100	0.0145

TABLE A6 REPLENISHMENT FREQUENCY

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, STATIONARY MODEL  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C(OUT)	C(FIX)	DP	PA	(24,24)
		/C(IN)	/C(IN)			
2	8	4	32	0.267	0.276	0.283
4	8	4	32	0.250	0.276	0.275
2	16	4	32	0.381	0.390	0.394
4	16	4	32	0.356	0.381	0.380
2	8	99	32	0.286	0.276	0.282
4	8	99	32	0.276	0.276	0.274
2	16	99	32	0.421	0.390	0.392
4	16	99	32	0.400	0.381	0.379
2	8	4	64	0.200	0.211	0.210
4	8	4	64	0.191	0.200	0.205
2	16	4	64	0.281	0.296	0.299
4	16	4	64	0.271	0.286	0.289
2	8	99	64	0.216	0.211	0.209
4	8	99	64	0.211	0.200	0.203
2	16	99	64	0.308	0.296	0.297
4	16	99	64	0.302	0.286	0.287

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, STATIONARY MODEL

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)  
 COLUMN (1) EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST  
 + : BIAS FOR O.C.: POSITIVE : - : NEGATIVE : \* : SIGNIFICANTLY POSITIVE : = : SIGNIFICANTLY NEGATIVE  
 SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT PERQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

		(1)		(2)	
		(24,24,24)		(24,24,24)	
LEADTIME	MEAN	C(OUT)	C(PIX)		
		/C(IN)	/C(IN)		
2	8	4	32	1.2	#
4	8	4	32	2.1	+
2	16	4	32	1.5	+
4	16	4	32	3.2	+
2	8	99	32	5.4	+
4	8	99	32	8.2	+
2	16	99	32	5.9	+
4	16	99	32	10.4	+
2	8	4	64	0.8	+
4	8	4	64	2.0	+
2	16	4	64	1.4	+
4	16	4	64	3.0	+
2	8	99	64	4.0	+
4	8	99	64	8.5	+
2	16	99	64	5.6	+
4	16	99	64	10.4	+

TABLE A8 ESTIMATED STANDARD DEVIATION OF FORECAST OF TOTAL COST

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, STATIONARY MODEL

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X, Y, Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR  
 REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	(24, 24, 24)
2	8	4	32	3.4
4	8	4	32	5.0
2	16	4	32	4.0
4	16	4	32	6.0
2	8	99	32	8.3
4	8	99	32	9.4
2	16	99	32	11.0
4	16	99	32	13.6
2	8	4	64	3.7
4	8	4	64	5.3
2	16	4	64	4.1
4	16	4	64	6.0
2	8	99	64	12.1
4	8	99	64	10.0
2	16	99	64	14.6
4	16	99	64	13.1



















TABLE A10 STANDARD DEVIATION OF BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, STATIONARY MODEL.  
STATISTICAL POLICES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVERSE PARAMETERS

LOADING	MEAN	C(CUT) /C(IN)	C(FIX) /C(IN)	PERIOD IN CYCLE					
				1	2	3	4	5	6
2	8	4	32	4.4	4.4	4.4	4.4	4.4	4.4
4	8	4	32	6.4	6.4	6.4	6.4	6.4	6.4
2	8	4	64	4.7	4.7	4.7	4.7	4.7	4.7
4	8	4	64	6.7	6.7	6.7	6.7	6.7	6.7
2	8	99	32	7.4	7.4	7.4	7.4	7.4	7.4
4	8	99	32	10.2	10.2	10.2	10.2	10.2	10.2
2	8	99	64	7.6	7.6	7.6	7.6	7.6	7.6
4	8	99	64	10.2	10.2	10.2	10.2	10.2	10.2
2	16	4	32	5.6	5.6	5.6	5.6	5.6	5.6
4	16	4	32	8.5	8.5	8.5	8.5	8.5	8.5
2	16	4	64	5.8	5.8	5.8	5.8	5.8	5.8
4	16	4	64	8.7	8.7	8.7	8.7	8.7	8.7
2	16	99	32	9.2	9.2	9.2	9.2	9.2	9.2
4	16	99	32	12.7	12.7	12.7	12.7	12.7	12.7
2	16	99	64	9.3	9.3	9.3	9.3	9.3	9.3
4	16	99	64	12.7	12.7	12.7	12.7	12.7	12.7



## Appendix B

### Single-item Data

#### Model I

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

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A8 Estimated Standard Deviation of Forecast of Total Cost	B7
A9 Values for $(s_i, S_i)$	B8 to B13
A10 Standard Deviations of $(s_i, S_i)$ Values	B14 to B15

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

TABLE A1 AVERAGE TOTAL COST

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL DP ARE % EXCESS OVER DP VALUE

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	DP	PA	(24,24)
2	8	4	32	25.3	1.1	4.3
4	8	4	32	28.0	0.9	6.7
2	16	4	32	35.2	1.1	3.9
4	16	4	32	39.1	0.9	6.0
2	8	99	32	41.5	1.4	10.4
4	8	99	32	47.4	1.1	13.8
2	16	99	32	56.6	1.9	10.3
4	16	99	32	65.0	1.2	12.6
2	8	4	64	32.2	1.8	4.5
4	8	4	64	34.6	1.8	5.8
2	16	4	64	45.6	1.0	3.5
4	16	4	64	48.9	1.1	4.4
2	8	99	64	49.0	2.2	10.4
4	8	99	64	54.8	1.6	11.7
2	16	99	64	67.8	1.6	8.6
4	16	99	64	75.9	1.2	10.3

TABLE A3 PERIOD-END INVENTORY

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MPAN	C(OUT)	C(FIX)	DP	PA	(24,24)
		/C(IN)	/C(IN)			
2	8	4	32	11.5	10.8	11.8
4	8	"	32	13.4	12.8	14.3
2	16	4	32	16.0	16.5	16.7
4	16	"	32	18.8	18.7	20.5
2	8	99	32	27.7	29.3	31.5
4	8	99	32	33.2	35.2	37.3
2	16	99	32	37.7	40.3	41.6
4	16	99	32	45.5	47.7	50.8
2	8	4	64	13.9	13.1	14.0
4	8	"	64	15.8	14.6	15.8
2	16	"	64	19.2	19.3	20.1
4	16	"	64	21.7	21.1	23.0
2	8	99	64	31.3	32.3	34.1
4	8	99	64	36.7	37.3	39.6
2	16	99	64	41.5	44.9	46.9
4	16	99	64	49.1	51.4	54.5

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	DP	PA	(24, 24)
2	8	4	32	0.1750	0.1834	0.1784
4	8	4	32	0.2088	0.2148	0.2162
2	16	4	32	0.1177	0.1094	0.1163
4	16	4	32	0.1409	0.1391	0.1421
2	8	99	32	0.0058	0.0051	0.0072
4	8	99	32	0.0068	0.0055	0.0105
2	16	99	32	0.0036	0.0035	0.0054
4	16	99	32	0.0044	0.0040	0.0068
2	8	4	64	0.1866	0.2072	0.2013
4	8	4	64	0.2247	0.2395	0.2435
2	16	4	64	0.1281	0.1261	0.1295
4	16	4	64	0.1542	0.1568	0.1541
2	8	99	64	0.0058	0.0061	0.0089
4	8	99	64	0.0069	0.0071	0.0112
2	16	99	64	0.0038	0.0035	0.0052
4	16	99	64	0.0045	0.0046	0.0070



TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
( $\bar{X}, Y$ ) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	DP	PA	(24, 24)
2	8	4	32	0.1901	0.1987	0.1894
4	8	4	32	0.1908	0.1964	0.1899
2	16	4	32	0.1926	0.1796	0.1848
4	16	4	32	0.1919	0.1891	0.1848
2	8	99	32	0.0091	0.0081	0.0105
4	8	99	32	0.0092	0.0074	0.0127
2	16	99	32	0.0092	0.0086	0.0121
4	16	99	32	0.0094	0.0086	0.0127
2	8	4	64	0.1904	0.2059	0.1965
4	8	4	64	0.1921	0.2027	0.1988
2	16	4	64	0.1923	0.1893	0.1895
4	16	4	64	0.1947	0.1974	0.1883
2	8	99	64	0.0090	0.0093	0.0125
4	8	99	64	0.0091	0.0093	0.0133
2	16	99	64	0.0094	0.0086	0.0115
4	16	99	64	0.0094	0.0094	0.0128

TABLE A6 REPLENISHMENT FREQUENCY

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C (OUT)	C (FIX)	DP	PA	(24,24)
		/C (IN)	/C (IN)			
2	8	4	32	0.254	0.276	0.275
4	8	4	32	0.247	0.268	0.270
2	16	4	32	0.366	0.378	0.388
4	16	"	32	0.352	0.369	0.369
2	8	99	32	0.290	0.271	0.270
4	8	99	32	0.279	0.265	0.262
2	16	99	32	0.410	0.370	0.381
4	16	99	32	0.391	0.364	0.362
2	8	4	64	0.192	0.205	0.206
4	8	4	64	0.181	0.201	0.202
2	16	4	64	0.283	0.291	0.293
4	16	4	64	0.270	0.285	0.284
2	8	99	64	0.206	0.202	0.202
4	8	99	64	0.198	0.199	0.199
2	16	99	64	0.319	0.287	0.289
4	16	99	64	0.307	0.282	0.281

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I

STATISTICAL POLICIES COMPUTED USING EXPRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y, Z) = (\text{REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST})$   
 COLUMN (1)  $\frac{1}{2}$  EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST  
 COLUMN (2)  $+$  : BIAS FOR O.C. : POSITIVE ;  $-$  : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE  
 SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

LEADTIME	MEAN	C(OUT)	C(PIX)	(1)		(2)	
				(24, 24, 24)	(24, 24, 24)	(24, 24, 24)	(24, 24, 24)
2	8	4	32	0.8	#	+	#
4	8	4	32	2.2	#	+	-
2	16	4	32	1.2	#	+	-
4	16	4	32	3.1	#	+	-
2	8	99	32	4.7	#	+	+
4	8	99	32	8.8	#	+	+
2	16	99	32	5.5	#	+	+
4	16	99	32	10.4	#	+	+
2	8	4	64	1.0	#	+	-
4	8	4	64	1.7	#	+	-
2	16	4	64	1.2	#	+	-
4	16	4	64	2.3	#	+	-
2	8	99	64	5.2	#	+	-
4	8	99	64	8.3	#	+	-
2	16	99	64	5.9	#	+	+
4	16	99	64	10.5	#	+	+

TABLE A3 ESTIMATED STANDARD DEVIATION OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR  
 REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	(24, 24, 24)
2	8	4	32	3.6
4	8	4	32	5.0
2	16	4	32	4.0
4	16	4	32	6.3
2	8	99	32	9.9
4	8	99	32	9.3
2	16	99	32	12.0
4	16	99	32	11.7
2	8	4	64	4.0
4	8	4	64	4.8
2	16	4	64	4.1
4	16	4	64	6.2
2	8	99	64	9.2
4	8	99	64	9.6
2	16	99	64	12.8
4	16	99	64	12.9



TABLE 29 VALUES FOR LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL 1  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	C(IN)	C(IN)/C(FIX)	PERIOD IN CYCLE											
					1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	D2 PA (24,24)		15	15	15	15	15	15	14	18	27	36	29	22
					15	15	15	15	15	15	15	21	27	34	27	21
					13	13	13	14	14	13	8	22.0	28.2	34.4	28.2	22.0
					12	12	12	12	12	12	12	16	25	33	26	19
8	4	64	D2 PA (24,24)		12	12	12	12	12	12	12	18	24	30	24	18
					12	12	12	12	12	12	12	18	24	30	24	18
					12	12	12	12	12	12	12	18	24	30	24	18
					12	12	12	12	12	12	12	18	24	30	24	18
8	99	32	D2 PA (24,24)		33	33	33	33	33	33	32	39	49	59	50	42
					33	33	33	33	33	33	32	39	49	59	50	42
					33	33	33	33	33	33	32	39	49	59	50	42
					33	33	33	33	33	33	32	39	49	59	50	42
8	99	64	D2 PA (24,24)		32	32	32	32	32	32	29	38	48	57	49	40
					31	31	31	31	31	31	31	39	47	55	47	39
					32	32	32	32	32	32	29	38	48	57	49	40
					32	32	32	32	32	32	29	38	48	57	49	40
16	4	32	D2 PA (24,24)		32	32	32	32	32	32	32	40.2	48.2	55.9	48.2	40.2
					33	33	33	33	33	33	32	44	55	74	60	46
					34	34	34	34	34	34	34	46	58	71	58	46
					34	34	34	34	34	34	34	46	58	71	58	46
16	4	64	D2 PA (24,24)		30	30	30	30	30	30	30	36	52	70	56	43
					30	30	30	30	30	30	30	36	52	70	56	43
					30	30	30	30	30	30	30	36	52	70	56	43
					30	30	30	30	30	30	30	36	52	70	56	43
16	99	32	D2 PA (24,24)		30	30	30	30	30	30	30	42.1	54.3	66.6	54.3	42.1
					30	30	30	30	30	30	30	42.1	54.3	66.6	54.3	42.1
					30	30	30	30	30	30	30	42.1	54.3	66.6	54.3	42.1
					30	30	30	30	30	30	30	42.1	54.3	66.6	54.3	42.1
16	99	64	D2 PA (24,24)		57	57	57	57	57	57	55	72	86	105	89	73
					57	57	57	57	57	57	55	72	86	105	89	73
					57	57	57	57	57	57	55	72	86	105	89	73
					57	57	57	57	57	57	55	72	86	105	89	73
16	99	32	D2 PA (24,24)		58.4	58.4	58.4	58.4	58.4	58.4	58.4	73.9	89.1	104.0	89.1	73.9
					58.4	58.4	58.4	58.4	58.4	58.4	58.4	73.9	89.1	104.0	89.1	73.9
					58.4	58.4	58.4	58.4	58.4	58.4	58.4	73.9	89.1	104.0	89.1	73.9
					58.4	58.4	58.4	58.4	58.4	58.4	58.4	73.9	89.1	104.0	89.1	73.9
16	99	64	D2 PA (24,24)		55	55	55	55	55	55	55	68	85	103	87	71
					55	55	55	55	55	55	55	68	85	103	87	71
					55	55	55	55	55	55	55	68	85	103	87	71
					55	55	55	55	55	55	55	68	85	103	87	71
16	99	64	D2 PA (24,24)		55.6	55.6	55.6	55.6	55.6	55.6	55.6	70.8	85.6	100.2	85.6	70.8
					55.6	55.6	55.6	55.6	55.6	55.6	55.6	70.8	85.6	100.2	85.6	70.8
					55.6	55.6	55.6	55.6	55.6	55.6	55.6	70.8	85.6	100.2	85.6	70.8
					55.6	55.6	55.6	55.6	55.6	55.6	55.6	70.8	85.6	100.2	85.6	70.8

TABLE A9 VALUES FOR BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	C(IN)	PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	37	38	38	37	36	37	40	53	57	57	50	44
			PA	35	35	35	35	35	35	35	44	53	52	53	44
			(24,24)	45	36.5	36.5	36.5	36.5	36.5	36.5	45.4	54.4	52.5	54.1	45.4
			DP	41	41	43	41	40	40	46	63	64	63	58	50
8	4	64	PA	41	41	41	41	41	41	41	51	61	70	61	51
			(24,24)	42.2	42.2	42.2	42.2	42.2	42.2	42.2	52.1	61.4	70.5	61.4	52.1
			DP	52	53	53	52	50	54	55	61	77	75	67	59
			PA	53	53	53	53	53	53	53	64	75	85	75	64
8	99	32	(24,24)	54.8	54.8	54.8	54.8	54.8	54.8	54.8	66.0	76.6	86.7	76.6	66.0
			DP	61	61	59	57	56	57	60	84	82	80	72	66
			PA	60	60	60	60	60	60	60	72	84	95	84	72
			(24,24)	61.5	61.5	61.5	61.5	61.5	61.5	61.5	73.7	85.1	96.0	85.1	73.7
16	4	32	DP	63	63	61	65	62	56	66	80	103	104	89	75
			PA	62	62	62	62	62	62	62	78	93	109	93	78
			(24,24)	62.2	62.2	62.2	62.2	62.2	62.2	62.2	78.5	93.9	102.3	93.9	78.5
			DP	71	72	74	72	68	71	80	81	113	112	98	85
16	4	64	PA	70	70	70	70	70	70	70	86	103	120	133	86
			(24,24)	70.2	70.2	70.2	70.2	70.2	70.2	70.2	87.6	104.5	121.0	104.5	87.6
			DP	82	83	82	82	84	77	67	108	130	128	112	95
			PA	85	85	85	85	85	85	85	104	122	140	122	104
16	99	32	(24,24)	86.5	86.5	86.5	86.5	86.5	86.5	86.5	105.8	123.7	131.2	123.7	105.8
			DP	92	90	97	93	87	78	94	109	139	136	120	106
			PA	95	95	95	95	95	95	95	114	134	152	134	114
			(24,24)	95.7	95.7	95.7	95.7	95.7	95.7	95.7	116.3	135.9	154.7	135.9	116.3

TABLE A9 VALUES FOR BIG S - LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	1	2	3	4	5	6	7	8	9	10	11	12
3	4	32	DP	22	23	23	21	22	26	35	30	21	21	22
			PA	20	20	20	20	20	20	23	26	23	26	23
			(24,24)	20.6	20.6	20.6	20.6	20.6	20.6	23.4	25.9	28.1	25.9	23.4
8	4	64	DP	32	32	30	27	27	38	47	39	30	30	31
			PA	29	29	29	29	29	29	33	37	40	37	33
			(24,24)	29.4	29.4	29.4	29.4	29.4	29.4	33.5	37.0	40.1	37.0	33.5
8	99	32	DP	19	20	20	19	17	23	22	28	16	17	17
			PA	20	20	20	20	20	20	23	26	26	25	23
			(24,24)	20.6	20.6	20.6	20.6	20.6	20.6	23.4	25.9	28.1	25.9	23.4
8	99	64	DP	29	29	27	25	24	33	46	34	23	23	26
			PA	29	29	29	29	29	29	33	37	40	37	33
			(24,24)	29.4	29.4	29.4	29.4	29.4	29.4	33.5	37.0	40.1	37.0	33.5
16	4	32	DP	30	30	28	33	28	37	36	43	30	29	29
			PA	28	28	28	28	28	28	32	35	38	35	32
			(24,24)	28.1	28.1	28.1	28.1	28.1	28.1	31.9	34.8	30.5	34.8	31.9
16	4	64	DP	41	43	45	42	37	53	45	61	42	42	42
			PA	40	40	40	40	40	40	45	50	54	50	45
			(24,24)	40.0	40.0	40.0	40.0	40.0	40.0	45.5	50.2	54.4	50.2	45.5
16	99	32	DP	25	26	25	25	27	18	12	44	23	23	22
			PA	28	28	28	28	28	28	32	35	38	35	32
			(24,24)	28.0	28.0	28.0	28.0	28.0	28.0	31.9	34.6	27.2	34.6	31.9
16	99	64	DP	37	35	42	38	33	41	41	54	33	33	35
			PA	40	40	40	40	40	40	45	50	54	50	45
			(24,24)	40.0	40.0	40.0	40.0	40.0	40.0	45.5	50.2	54.4	50.2	45.5

TABLE A9 VALUES FOR LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I  
 REPLENISHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN C(OUT) C(FIX) /C(IN) /C(IN)		PERIOD IN CYCLE											
		1	2	3	4	5	6	7	8	9	10	11	12
3	4	32	29	29	29	28	32	39	50	50	49	42	35
		LP	29	29	29	29	35	42	48	48	48	42	35
		(24,24)	30.0	30.0	30.0	30.0	36.4	42.8	49.2	49.2	49.2	42.8	36.4
8	4	64	27	29	26	22	28	37	47	47	46	39	32
		DP	27	29	26	26	32	38	44	44	44	38	32
		(24,24)	26.6	26.6	26.6	26.6	32.6	38.8	45.0	45.0	45.0	38.8	32.6
8	99	32	51	51	51	49	56	65	75	76	75	67	58
		DP	51	51	51	51	59	67	74	74	74	67	59
		(24,24)	52.1	52.1	52.1	52.1	60.2	68.1	76.0	76.0	76.0	68.1	60.2
8	99	64	49	49	49	47	54	63	73	74	73	65	57
		DP	49	49	49	49	56	64	71	71	71	64	56
		(24,24)	49.4	49.4	49.4	49.4	57.3	65.1	72.7	72.7	72.7	65.1	57.3
16	4	32	61	61	61	55	71	81	100	100	100	87	74
		DP	61	61	61	61	73	86	98	98	98	86	73
		(24,24)	61.6	61.6	61.6	61.6	74.5	87.3	100.1	100.1	100.1	87.3	74.5
16	4	64	56	57	57	53	63	77	96	96	96	83	68
		DP	56	57	57	56	68	81	93	93	93	81	68
		(24,24)	57.1	57.1	57.1	57.1	69.5	82.1	94.7	94.7	94.7	82.1	69.5
16	99	32	89	89	89	86	103	116	134	135	135	120	104
		DP	89	89	89	89	104	119	133	133	133	119	104
		(24,24)	91.1	91.1	91.1	91.1	106.2	121.0	135.8	135.8	135.8	121.0	106.2
16	99	64	86	87	88	86	99	114	132	133	133	118	102
		DP	86	87	88	86	100	115	129	129	129	115	100
		(24,24)	87.4	87.4	87.4	87.4	102.3	116.9	131.4	131.4	131.4	116.9	102.3





TABLE A9 VALUES FOR BIG S - LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I  
 REPLENISHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = DIVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	C(IN)	PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	24	23	22	23	27	27	35	24	23	23	23	24
			PA	21	21	21	21	21	23	25	26	26	26	25	23
			(24,24)	21.3	21.3	21.3	21.3	21.3	23.1	24.9	26.3	26.3	26.3	24.9	23.1
8	4	64	DP	31	30	27	30	36	52	43	32	31	32	33	34
			PA	30	30	30	30	30	33	35	37	37	37	35	33
			(24,24)	30.5	30.5	30.5	30.5	30.5	33.1	35.5	37.7	37.7	37.7	35.5	33.1
8	99	32	DP	21	19	17	20	28	22	31	21	18	17	17	19
			PA	21	21	21	21	21	23	25	26	26	26	25	23
			(24,24)	21.3	21.3	21.3	21.3	21.3	23.1	24.9	26.3	26.3	26.3	24.9	23.1
8	99	64	DP	28	27	26	27	33	49	40	28	25	25	26	28
			PA	30	30	30	30	30	33	35	37	37	37	35	33
			(24,24)	30.5	30.5	30.5	30.5	30.5	33.1	35.5	37.7	37.7	37.7	35.5	33.1
16	4	32	DP	30	35	30	22	39	36	49	34	33	32	31	30
			PA	29	29	29	29	29	31	34	36	36	36	34	31
			(24,24)	29.1	29.1	29.1	29.1	29.1	31.6	33.8	35.9	35.9	35.9	33.8	31.6
16	4	64	DP	47	42	39	44	56	45	66	47	46	43	42	46
			PA	41	41	41	41	41	45	48	51	51	51	48	45
			(24,24)	41.5	41.5	41.5	41.5	41.5	45.1	48.2	51.1	51.1	51.1	48.2	45.1
16	99	32	DP	25	23	28	18	14	36	44	30	28	25	24	25
			PA	29	29	29	29	29	31	34	36	36	36	34	31
			(24,24)	29.1	29.1	29.1	29.1	29.1	31.6	33.8	35.9	35.9	35.9	33.8	31.6
16	99	64	DP	44	39	31	25	41	41	60	42	38	35	33	35
			PA	41	41	41	41	41	45	48	51	51	51	48	45
			(24,24)	41.5	41.5	41.5	41.5	41.5	45.1	48.2	51.1	51.1	51.1	48.2	45.1

TABLE A10 STANDARD DEVIATION OF LITTLE'S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) ; X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	32	2.4	2.4	2.4	2.4	2.4	2.4	2.4	3.2	4.0	4.8	4.0	3.2
4	8	4	32	4.1	4.1	4.1	4.1	4.1	4.9	5.7	6.5	6.5	6.5	5.7	4.9
2	8	4	64	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.9	3.6	4.4	3.6	2.9
4	8	4	64	3.7	3.7	3.7	3.7	3.7	4.6	5.3	6.1	6.1	6.1	5.3	4.6
2	8	99	32	5.4	5.4	5.4	5.4	5.4	5.4	5.4	6.5	7.5	8.5	7.5	6.5
4	8	99	32	7.5	7.5	7.6	7.6	7.6	8.5	9.5	10.5	10.5	10.5	9.5	8.5
2	8	99	64	4.9	4.9	4.9	4.9	4.9	4.9	4.9	6.0	6.9	7.9	6.9	6.0
4	8	99	64	7.0	7.0	7.0	7.0	7.0	8.0	8.9	9.8	9.8	9.8	8.9	8.0
2	16	4	32	3.3	3.3	3.3	3.3	3.3	3.3	3.3	4.5	5.7	6.8	5.7	4.5
4	16	4	32	5.8	5.8	5.8	5.8	5.8	6.9	8.0	9.2	9.2	9.2	8.0	6.9
2	16	4	64	2.2	2.2	2.2	2.2	2.2	2.2	2.2	3.2	4.3	5.3	4.3	3.2
4	16	4	64	5.4	5.4	5.4	5.4	5.4	6.6	7.7	8.8	8.8	8.8	7.7	6.6
2	16	99	32	6.8	6.8	6.8	6.8	6.8	6.8	6.8	8.2	9.5	10.9	9.5	8.2
4	16	99	32	9.6	9.6	9.6	9.6	9.6	10.9	12.3	13.5	13.5	13.5	12.3	10.9
2	16	99	64	6.2	6.2	6.2	6.2	6.2	6.2	6.2	7.5	8.8	10.2	8.8	7.5
4	16	99	64	9.0	9.0	9.0	9.0	9.0	10.3	11.5	12.7	12.7	12.7	11.5	10.3

TABLE A10 STANDARD DEVIATION OF BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL I  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

LEADTIME	MEAN	C(OUT) /C(IN)	C(PIX) /C(IN)	1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	32	3.7	3.7	3.7	3.7	3.7	3.7	3.7	4.7	5.6	6.5	5.6	4.7
4	8	4	32	5.4	5.4	5.4	5.4	5.4	6.3	7.2	8.1	8.1	8.1	7.2	6.3
2	8	4	64	3.9	3.9	3.9	3.9	3.9	3.9	3.9	4.9	5.9	6.9	5.9	4.9
4	8	4	64	5.6	5.6	5.6	5.6	5.6	6.6	7.5	8.4	8.4	8.4	7.5	6.6
2	8	99	32	6.7	6.7	6.7	6.7	6.7	6.7	6.7	7.9	9.2	10.3	9.2	7.9
4	8	99	32	8.8	8.8	8.8	8.8	8.8	10.0	11.0	12.1	12.1	12.1	11.0	10.0
2	8	99	64	6.7	6.7	6.7	6.7	6.7	6.7	6.7	8.0	9.3	10.4	9.3	8.0
4	8	99	64	8.8	8.8	8.8	8.8	8.8	10.0	11.1	12.2	12.2	12.2	11.1	10.0
2	16	4	32	4.6	4.6	4.6	4.6	4.6	4.6	4.6	5.9	7.0	8.0	7.0	5.9
4	16	4	32	7.1	7.1	7.1	7.1	7.1	8.3	9.5	10.8	10.8	10.8	9.5	8.3
2	16	4	64	4.9	4.9	4.9	4.9	4.9	4.9	4.9	6.3	7.5	8.9	7.5	6.3
4	16	4	64	7.2	7.2	7.2	7.2	7.2	8.6	9.8	11.0	11.0	11.0	9.8	8.6
2	16	99	32	8.1	8.1	8.1	8.1	8.1	8.1	8.1	9.7	11.3	14.5	11.3	9.7
4	16	99	32	11.0	11.0	11.0	11.0	11.0	12.4	13.8	15.2	15.2	15.2	13.8	12.4
2	16	99	64	8.1	8.1	8.1	8.1	8.1	8.1	8.1	9.7	11.2	12.6	11.2	9.7
4	16	99	64	10.9	10.9	10.9	10.9	10.9	12.4	13.7	15.1	15.1	15.1	13.7	12.4



## Appendix C

### Single-item Data

#### Model II

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

	<u>page</u>
Table A1 Average Total Cost	C1
A3 Period-End Inventory	C2
A4 Period-End Backlog as Proportion of Mean Demand	C3
A5 Frequency of Periods with Backlog	C4
A6 Replenishment Frequency	C5
A7 Estimated Bias of Forecast of Total Cost	C6
A8 Estimated Standard Deviation of Forecast of Total Cost	C7
A9 Values for $(s_i, S_i)$	C8 to C13
A10 Standard Deviations of $(s_i, S_i)$ Values	C14 to C15

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

TABLE A1 AVERAGE TOTAL COST

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL DP ARE % EXCESS OVER DP VALUE

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	DP	PA	(24,24)
2	8	4	32	25.3	0.7	3.8
4	8	4	32	28.1	0.7	6.7
2	16	4	32	35.6	0.3	3.2
4	16	4	32	39.4	0.4	5.5
2	8	99	32	41.7	0.7	9.9
4	8	99	32	47.6	0.8	13.2
2	16	99	32	57.1	0.7	9.2
4	16	99	32	65.4	0.6	12.1
2	8	4	64	32.4	1.1	3.3
4	8	4	64	34.8	1.2	5.2
2	16	4	64	45.8	0.5	2.5
4	16	4	64	49.1	0.7	4.7
2	8	99	64	49.4	1.3	8.0
4	8	99	64	55.1	1.1	12.5
2	16	99	64	68.4	0.8	6.9
4	16	99	64	76.5	0.8	10.1

TABLE A3 PERIOD-END INVENTORY

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	DP	PA	(24,24)
2	8	4	32	11.4	10.6	11.9
4	8	4	32	13.4	12.8	14.3
2	16	4	32	16.0	16.2	16.9
4	16	4	32	18.8	18.7	20.3
2	8	99	32	28.0	28.9	30.9
4	8	99	32	33.5	35.1	37.3
2	16	99	32	38.1	39.7	41.7
4	16	99	32	45.8	47.8	50.3
2	8	4	64	14.1	12.9	13.8
4	8	4	64	15.8	14.6	15.9
2	16	4	64	19.7	18.9	19.9
4	16	4	64	22.3	21.0	22.6
2	8	99	64	31.5	32.2	33.7
4	8	99	64	36.6	37.5	39.7
2	16	99	64	42.6	44.3	46.1
4	16	99	64	50.0	51.2	53.7

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y) = (\text{REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION})$

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	DP	PA	(24, 24)
2	8	4	32	0.1743	0.1870	0.1731
4	8	4	32	0.2112	0.2151	0.2188
2	16	4	32	0.1176	0.1120	0.1150
4	16	4	32	0.1440	0.1387	0.1449
2	8	99	32	0.0057	0.0054	0.0078
4	8	99	32	0.0069	0.0056	0.0102
2	16	99	32	0.0037	0.0035	0.0053
4	16	99	32	0.0043	0.0039	0.0071
2	8	4	64	0.1879	0.2092	0.2034
4	8	4	64	0.2231	0.2393	0.2443
2	16	4	64	0.1320	0.1312	0.1297
4	16	4	64	0.1531	0.1593	0.1654
2	8	99	64	0.0058	0.0060	0.0083
4	8	99	64	0.0069	0.0069	0.0120
2	16	99	64	0.0037	0.0038	0.0053
4	16	99	64	0.0046	0.0048	0.0078



TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y) = (\text{REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION})$

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	DP	PA	(24, 24)
2	8	4	32	0.1895	0.2021	0.1847
4	8	4	32	0.1920	0.1966	0.1906
2	16	4	32	0.1910	0.1833	0.1833
4	16	4	32	0.1943	0.1890	0.1881
2	8	99	32	0.0090	0.0084	0.0113
4	8	99	32	0.0092	0.0075	0.0123
2	16	99	32	0.0094	0.0088	0.0119
4	16	99	32	0.0093	0.0083	0.0131
2	8	4	64	0.1901	0.2073	0.1990
4	8	4	64	0.1908	0.2025	0.1995
2	16	4	64	0.1944	0.1946	0.1891
4	16	4	64	0.1921	0.1995	0.1982
2	8	99	64	0.0090	0.0092	0.0118
4	8	99	64	0.0092	0.0090	0.0140
2	16	99	64	0.0092	0.0091	0.0117
4	16	99	64	0.0095	0.0098	0.0140

TABLE A6 REPLENISHMENT FREQUENCY

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	DP	PA	(24, 24)
2	8	4	32	0.260	0.279	0.277
4	8	4	32	0.246	0.268	0.270
2	16	4	32	0.377	0.385	0.389
4	16	4	32	0.356	0.375	0.376
2	8	99	32	0.286	0.276	0.272
4	8	99	32	0.274	0.266	0.266
2	16	99	32	0.408	0.380	0.382
4	16	99	32	0.399	0.370	0.369
2	8	4	64	0.193	0.206	0.207
4	8	4	64	0.185	0.202	0.202
2	16	4	64	0.277	0.293	0.293
4	16	4	64	0.266	0.286	0.286
2	8	99	64	0.208	0.204	0.204
4	8	99	64	0.204	0.200	0.201
2	16	99	64	0.311	0.291	0.291
4	16	99	64	0.300	0.285	0.284

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)  
 COLUMN (1) X EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST  
 COLUMN (2) + : BIAS FOR O.C.: POSITIVE : - : NEGATIVE : # : SIGNIFICANTLY POSITIVE : = : SIGNIFICANTLY NEGATIVE  
 SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

LEADTIME	MEAN	(1)		(2)	
		C(OUT) /C(IN)	C(FIX) /C(IN)	(24,24,24)	(24,24,24)
2	8	4	32	+	+
4	8	4	32	+	+
2	16	4	32	+	+
4	16	4	32	+	+
2	8	99	32	+	+
4	8	99	32	+	+
2	16	99	32	+	+
4	16	99	32	+	+
2	8	4	64	+	+
4	8	4	64	+	+
2	16	4	64	+	+
4	16	4	64	+	+
2	8	99	64	+	+
4	8	99	64	+	+
2	16	99	64	+	+
4	16	99	64	+	+

TABLE A8 ESTIMATED STANDARD DEVIATION OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y, Z)$  = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR  
 REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	(24, 24, 24)
2	8	4	32	3.4
4	8	4	32	5.0
2	16	4	32	4.3
4	16	4	32	6.1
2	8	99	32	9.3
4	8	99	32	10.2
2	16	99	32	11.0
4	16	99	32	16.7
2	8	4	64	3.9
4	8	4	64	5.2
2	16	4	64	4.4
4	16	4	64	6.3
2	8	99	64	10.5
4	8	99	64	10.0
2	16	99	64	12.8
4	16	99	64	16.6



TABLE A9 VALUES FOR LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP (24,24)	15 15 15.9	16 15 15.9	15 15 15.9	15 15 15.9	16 17 17.9	19 21 21.9	27 27 28.1	31 30 30.2	29 27 28.1	22 21 21.9	18 17 17.9
8	4	64	DP (24,24)	13 12 12.9	14 12 12.9	14 12 12.9	14 12 12.9	12 14 14.7	16 18 18.6	25 24 24.4	28 26 26.4	26 24 24.4	19 18 18.6	15 14 14.7
8	99	32	DP (24,24)	33 33 34.2	33 33 34.2	33 33 34.2	33 33 34.2	35 36 37.0	40 41 42.6	49 49 50.7	53 52 53.3	50 49 50.7	42 41 42.6	36 36 37.0
8	99	64	DP (24,24)	31 31 32.1	31 31 32.1	31 31 32.1	31 31 32.1	34 34 34.8	38 39 40.2	47 47 48.1	50 49 50.8	47 47 48.1	39 39 40.2	34 34 34.8
16	4	32	DP (24,24)	34 34 34.1	34 34 34.1	34 34 34.1	34 34 34.1	36 38 38.3	43 46 46.6	56 58 59.1	63 62 63.3	60 58 59.1	46 46 46.6	38 38 38.3
16	4	64	DP (24,24)	30 30 30.2	30 30 30.2	30 30 30.2	30 30 30.2	31 31 31.1	37 37 37.9	52 52 53.0	59 57 58.3	56 53 54.2	43 41 42.1	34 34 34.1
16	99	32	DP (24,24)	57 57 58.4	57 57 58.4	57 57 58.4	57 57 58.4	62 62 63.6	72 73 73.9	87 87 89.0	92 92 94.1	87 87 89.0	72 72 73.9	62 62 63.6
16	99	64	DP (24,24)	55 55 55.6	55 55 55.6	55 55 55.6	55 55 55.6	60 60 60.8	69 69 70.7	84 84 85.7	89 89 90.5	84 84 85.7	69 69 70.7	60 60 60.8

TABLE A9 VALUES FOR BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	C(IN)	PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	37	37	37	37	38	41	51	56	55	50	44	40
			PA	35	35	35	35	35	38	44	53	56	53	44	38
			(24,24)	36.5	36.5	36.5	36.5	36.5	39.5	45.3	54.0	56.9	54.0	45.3	39.5
8	4	64	DP	45	44	42	41	42	54	62	63	61	57	50	47
			PA	41	41	41	41	41	45	51	61	64	61	51	45
			(24,24)	42.3	42.3	42.3	42.3	42.3	45.6	52.0	61.4	64.4	61.4	52.0	45.6
8	99	32	DP	53	52	52	53	55	54	70	75	73	67	59	55
			PA	53	53	53	53	53	57	64	75	78	75	64	57
			(24,24)	54.8	54.8	54.8	54.8	54.8	58.6	66.0	76.6	80.0	76.6	66.0	58.6
8	99	64	DP	60	60	60	60	60	74	82	82	79	73	66	62
			PA	60	60	60	60	60	65	72	84	88	84	72	65
			(24,24)	61.6	61.6	61.6	61.6	61.6	65.7	73.7	85.1	88.8	85.1	73.7	65.7
16	4	32	DP	62	62	62	63	63	67	81	95	98	93	76	67
			PA	62	62	62	62	62	67	78	93	98	93	78	67
			(24,24)	62.2	62.2	62.2	62.2	62.2	67.7	78.5	93.8	98.1	93.8	78.5	67.7
16	4	64	DP	73	72	71	72	73	80	94	110	107	98	85	76
			PA	70	70	70	70	70	76	86	103	108	103	86	76
			(24,24)	70.2	70.2	70.2	70.2	70.2	76.1	87.6	104.5	110.0	104.5	87.6	76.1
16	99	32	DP	82	82	83	80	81	92	108	122	123	112	97	87
			PA	85	85	85	85	85	91	104	122	128	122	104	91
			(24,24)	86.5	86.5	86.5	86.5	86.5	93.1	105.8	123.5	128.3	123.5	105.8	93.1
16	99	64	DP	95	93	90	90	100	94	112	137	132	121	105	96
			PA	95	95	95	95	95	102	114	134	140	134	114	102
			(24,24)	95.7	95.7	95.7	95.7	95.7	102.7	116.2	135.9	142.2	135.9	116.2	102.7

TABLE A9 VALUES FOR BIG S - LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN C(OUT) C(FIX)		C(IN) /C(IN)		1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	22	22	21	22	23	25	32	29	24	21	22	22
			PA	20	20	20	20	20	21	23	26	26	26	23	21
			(24,24)	20.6	20.6	20.6	20.6	20.6	21.6	23.4	25.9	26.7	25.9	23.4	21.6
8	4	64	DP	32	31	28	27	30	42	46	38	33	31	31	32
			PA	29	29	29	29	29	31	33	37	38	37	33	31
			(24,24)	29.4	29.4	29.4	29.4	29.4	30.9	33.5	37.0	38.0	37.0	33.5	30.9
8	99	32	DP	20	19	19	20	22	19	30	26	20	17	17	19
			PA	20	20	20	20	20	21	23	26	26	26	23	21
			(24,24)	20.6	20.6	20.6	20.6	20.6	21.6	23.4	25.9	26.7	25.9	23.4	21.6
8	99	64	DP	28	28	27	27	26	42	44	34	27	24	26	27
			PA	29	29	29	29	29	31	33	37	38	37	33	31
			(24,24)	29.4	29.4	29.4	29.4	29.4	30.9	33.5	37.0	38.0	37.0	33.5	30.9
16	4	32	DP	29	30	30	27	31	31	38	39	35	29	29	29
			PA	28	28	28	28	28	29	32	35	35	35	32	29
			(24,24)	28.0	28.0	28.0	28.0	28.0	29.4	31.9	34.7	34.8	34.7	31.9	29.4
16	4	64	DP	43	42	41	43	44	49	57	58	48	42	42	42
			PA	40	40	40	40	40	42	45	50	51	50	45	42
			(24,24)	40.0	40.0	40.0	40.0	40.0	42.0	45.5	50.3	51.7	50.3	45.5	42.0
16	99	32	DP	25	26	26	23	25	31	37	35	30	23	24	24
			PA	28	28	28	28	28	29	32	35	36	35	32	29
			(24,24)	28.0	28.0	28.0	28.0	28.0	29.4	31.9	34.4	34.2	34.4	31.9	29.4
16	99	64	DP	40	38	34	35	46	35	44	52	40	34	33	35
			PA	40	40	40	40	40	42	45	50	51	50	45	42
			(24,24)	40.0	40.0	40.0	40.0	40.0	42.0	45.5	50.2	51.7	50.2	45.5	42.0

TABLE A9 VALUES FOR LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II  
 REPLENISHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,I) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(PIX) /C(IN)		PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	29	29	28	30	32	40	46	50	47	42	36	31
			PA	29	29	29	31	35	42	46	48	46	42	35	31
			(24,24)	30.0	30.0	30.0	32.1	36.3	42.7	47.0	49.1	47.0	42.7	36.3	32.1
8	4	64	DP	27	27	26	25	29	37	44	47	44	39	33	29
			PA	26	26	26	28	32	38	42	44	42	38	32	28
			(24,24)	26.5	26.5	26.5	28.5	32.6	38.8	42.9	44.9	42.9	38.8	32.6	28.5
8	99	32	DP	50	50	50	52	56	65	72	75	72	67	59	53
			PA	51	51	51	53	59	67	72	74	72	67	59	53
			(24,24)	52.0	52.0	52.0	54.8	60.1	68.1	73.3	75.9	73.3	68.1	60.1	54.8
8	99	64	DP	49	49	48	50	54	64	70	74	71	65	57	51
			PA	48	48	48	51	56	64	69	71	69	64	56	51
			(24,24)	49.4	49.4	49.4	52.0	57.2	65.0	70.2	72.7	70.2	65.0	57.2	52.0
16	4	32	DP	60	60	58	62	69	83	93	100	96	87	73	64
			PA	61	61	61	65	73	86	94	98	94	86	73	65
			(24,24)	61.5	61.5	61.5	65.8	74.3	87.1	95.7	100.0	95.7	87.1	74.3	65.8
16	4	64	DP	56	56	55	58	64	78	90	96	92	83	69	61
			PA	56	56	56	60	68	81	89	93	89	81	68	60
			(24,24)	57.0	57.0	57.0	61.2	69.5	82.0	90.3	94.5	90.3	82.0	69.5	61.2
16	99	32	DP	89	89	88	93	102	117	128	135	130	120	105	94
			PA	89	89	89	94	104	119	129	133	129	119	104	94
			(24,24)	91.0	91.0	91.0	96.0	106.0	120.9	130.8	135.7	130.8	120.9	106.0	96.0
16	99	64	DP	87	86	86	90	99	114	126	133	128	117	102	92
			PA	86	86	86	91	100	115	124	129	124	115	100	91
			(24,24)	87.4	87.4	87.4	92.3	102.2	116.8	126.5	131.3	126.5	116.8	102.2	92.3





TABLE A9 VALUES FOR BIG S - LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II  
 REPLENISHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	C(IN)	PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	23	23	25	26	32	32	28	23	23	23	23	23
			PA	21	21	21	22	23	25	26	26	26	25	23	22
			(24,24)	21.3	21.3	21.3	22.0	23.1	24.9	25.9	26.3	25.9	24.9	23.1	22.0
8	4	64	DP	30	30	31	39	49	42	35	32	32	33	33	32
			PA	30	30	30	31	33	35	37	37	37	35	33	31
			(24,24)	30.5	30.5	30.5	31.4	33.1	35.5	36.9	37.6	36.9	35.5	33.1	31.4
8	99	32	DP	20	21	22	24	27	30	23	19	18	18	18	20
			PA	21	21	21	22	23	25	26	26	26	25	23	22
			(24,24)	21.3	21.3	21.3	22.0	23.1	24.9	25.9	26.3	25.9	24.9	23.1	22.0
8	99	64	DP	28	28	29	33	37	38	31	26	25	26	27	29
			PA	30	30	30	31	33	35	37	37	37	35	33	31
			(24,24)	30.5	30.5	30.5	31.4	33.1	35.5	36.9	37.7	36.9	35.5	33.1	31.4
16	4	32	DP	31	30	35	33	39	39	40	33	32	31	31	32
			PA	29	29	28	30	31	34	35	36	35	34	31	30
			(24,24)	29.1	29.1	29.1	30.0	31.6	33.8	35.2	35.9	35.2	33.8	31.6	30.0
16	4	64	DP	43	43	46	52	57	62	52	45	40	44	45	44
			PA	41	41	41	42	45	48	50	51	50	48	45	42
			(24,24)	41.5	41.5	41.5	42.7	45.0	48.2	50.2	51.1	50.2	48.2	45.0	42.7
16	99	32	DP	27	24	26	31	37	34	34	27	25	24	24	27
			PA	29	29	29	30	31	34	35	36	35	34	31	30
			(24,24)	29.1	29.1	29.1	30.0	31.6	33.8	35.2	35.9	35.2	33.8	31.6	30.0
16	99	64	DP	37	38	45	35	45	57	47	38	36	36	37	38
			PA	41	41	41	42	45	48	50	51	50	48	45	42
			(24,24)	41.5	41.5	41.5	42.7	45.0	48.2	50.2	51.1	50.2	48.2	45.0	42.7

TABLE A10 STANDARD DEVIATION OF LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (I,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

LEADTIME	MEAN	C(OUT)	C(FIX)	PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	32	2.3	2.3	2.3	2.3	2.3	2.6	3.2	4.0	4.3	4.0	3.2	2.6
4	8	4	32	4.1	4.1	4.1	4.3	4.8	5.6	6.2	6.4	6.2	5.6	4.8	4.3
2	8	4	64	2.2	2.2	2.2	2.2	2.2	2.4	2.9	3.7	4.0	3.7	2.9	2.4
4	8	4	64	3.8	3.8	3.8	4.0	4.5	5.3	5.8	6.1	5.8	5.3	4.5	4.0
2	8	99	32	5.4	5.4	5.4	5.4	5.4	5.8	6.5	7.5	7.9	7.5	6.5	5.8
4	8	99	32	7.5	7.5	7.5	7.8	8.6	9.5	10.1	10.5	10.1	9.5	8.6	7.8
2	8	99	64	4.9	4.9	4.9	4.9	4.9	5.3	5.9	7.0	7.3	7.0	5.9	5.3
4	8	99	64	7.0	7.0	7.0	7.3	7.9	8.9	9.5	9.8	9.5	8.9	7.9	7.3
2	16	4	32	3.4	3.4	3.4	3.4	3.4	3.8	4.5	5.7	6.0	5.7	4.5	3.8
4	16	4	32	5.7	5.7	5.7	6.1	6.8	8.0	8.8	9.1	8.8	8.0	6.8	6.1
2	16	4	64	3.2	3.2	3.2	3.2	3.2	3.5	4.3	5.4	5.8	5.4	4.3	3.5
4	16	4	64	5.4	5.4	5.4	5.8	6.5	7.6	8.4	8.7	8.4	7.6	6.5	5.8
2	16	99	32	6.8	6.8	6.8	6.8	6.8	7.3	8.2	9.5	10.0	9.5	8.2	7.3
4	16	99	32	9.6	9.6	9.6	10.1	11.0	12.2	13.1	13.5	13.1	12.2	11.0	10.1
2	16	99	64	6.2	6.2	6.2	6.2	6.2	6.6	7.6	8.9	9.4	8.9	7.6	6.6
4	16	99	64	8.9	8.9	8.9	9.4	10.2	11.5	12.3	12.7	12.3	11.5	10.2	9.4

TABLE A10 STANDARD DEVIATION OF BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL II  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	32	3.7	3.7	3.7	3.7	3.7	4.0	4.7	5.6	6.0	5.6	4.7	4.0
4	8	4	32	5.4	5.4	5.4	5.7	6.3	7.2	7.8	8.0	7.8	7.2	6.3	5.7
2	8	4	64	4.0	4.0	4.0	4.0	4.0	4.2	5.0	6.0	6.3	6.0	5.0	4.2
4	8	4	64	5.6	5.6	5.6	5.9	6.5	7.4	8.0	8.4	8.0	7.4	6.5	5.9
2	8	99	32	6.6	6.6	6.6	6.6	6.6	7.1	7.9	9.2	9.6	9.2	7.9	7.1
4	8	99	32	8.8	8.8	8.8	9.2	10.0	11.0	11.7	12.1	11.7	11.0	10.0	9.2
2	8	99	64	6.8	6.8	6.8	6.8	6.8	7.2	8.1	9.3	9.7	9.3	8.1	7.2
4	8	99	64	8.8	8.8	8.8	9.2	10.0	11.1	11.7	12.1	11.7	11.1	10.0	9.2
2	16	4	32	4.7	4.7	4.7	4.7	4.7	5.1	6.0	6.9	6.8	6.9	6.0	5.1
4	16	4	32	7.0	7.0	7.0	7.5	8.2	9.5	10.4	10.7	10.4	9.5	8.2	7.5
2	16	4	64	4.9	4.9	4.9	4.9	4.9	5.3	6.3	7.6	8.0	7.6	6.3	5.3
4	16	4	64	7.2	7.2	7.2	7.6	8.5	9.7	10.5	10.9	10.5	9.7	8.5	7.6
2	16	99	32	8.1	8.1	8.1	8.1	8.1	8.6	9.7	11.2	11.8	11.2	9.7	8.6
4	16	99	32	11.0	11.0	11.0	11.5	12.4	13.8	14.8	15.2	14.8	13.8	12.4	11.5
2	16	99	64	8.1	8.1	8.1	8.1	8.1	8.6	9.7	11.3	11.8	11.3	9.7	8.6
4	16	99	64	10.9	10.9	10.9	11.3	12.3	13.7	14.6	15.1	14.6	13.7	12.3	11.3



## Appendix D

### Single-item Data

#### Model III

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

	<u>page</u>
Table A1 Average Total Cost	D1
A3 Period-End Inventory	D2
A4 Period-End Backlog as Proportion of Mean Demand	D3
A5 Frequency of Periods with Backlog	D4
A6 Replenishment Frequency	D5
A7 Estimated Bias of Forecast of Total Cost	D6
A8 Estimated Standard Deviation of Forecast of Total Cost	D7
A9 Values for $(s_i, S_i)$	D8 to D13
A10 Standard Deviations of $(s_i, S_i)$ Values	D14 to D15

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

TABLE A1 AVERAGE TOTAL COST

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL DP ARE % EXCESS OVER DP VALUE

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	DP	PA	(24,24)
2	8	4	32	24.4	2.6	7.5
4	8	4	32	27.1	3.5	10.7
2	16	4	32	34.1	6.2	9.2
4	16	4	32	38.0	4.1	9.2
2	8	99	32	40.1	5.5	16.6
4	8	99	32	46.2	7.0	23.9
2	16	99	32	54.3	7.5	18.6
4	16	99	32	62.7	7.8	18.7
2	8	4	64	31.0	5.1	9.9
4	8	4	64	33.3	5.4	10.7
2	16	4	64	43.4	2.5	4.9
4	16	4	64	46.8	3.9	9.1
2	8	99	64	47.1	8.7	19.4
4	8	99	64	52.8	4.9	17.6
2	16	99	64	64.4	5.4	13.1
4	16	99	64	72.4	8.0	21.4

TABLE A3 PERIOD-END INVENTORY

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y) = (\text{REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION})$

LEADTIME	MEAN	C (OUT)	C (FIX)	DP	PA	(24, 24)
		/C (IN)	/C (IN)			
2	8	4	32	11.1	12.1	13.1
4	8	4	32	13.3	13.8	15.6
2	16	4	32	15.7	13.9	14.9
4	16	4	32	19.0	20.6	20.8
2	8	99	32	27.3	31.3	33.5
4	8	99	32	33.2	37.9	41.3
2	16	99	32	36.0	36.3	38.8
4	16	99	32	44.1	52.1	52.6
2	8	4	64	14.1	14.9	16.1
4	8	4	64	16.1	16.3	17.3
2	16	4	64	19.2	19.7	20.7
4	16	4	64	22.0	21.8	24.3
2	8	99	64	30.6	36.5	39.3
4	8	99	64	36.6	39.4	42.7
2	16	99	64	41.1	45.4	47.3
4	16	99	64	49.5	54.2	58.4

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y) = (\text{REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION})$

LEADTIME	MEAN	C (OUT)	C (FIX)	DP	PA	(24, 24)
		/C (IN)	/C (IN)			
2	8	4	32	0.1635	0.1483	0.1566
4	8	4	32	0.2036	0.1921	0.2024
2	16	4	32	0.1117	0.1237	0.1278
4	16	4	32	0.1377	0.1127	0.1319
2	8	99	32	0.0057	0.0044	0.0072
4	8	99	32	0.0068	0.0050	0.0105
2	16	99	32	0.0035	0.0040	0.0065
4	16	99	32	0.0043	0.0026	0.0062
2	8	4	64	0.1806	0.1722	0.1780
4	8	4	64	0.2176	0.1998	0.2225
2	16	4	64	0.1191	0.1196	0.1209
4	16	4	64	0.1447	0.1555	0.1554
2	8	99	64	0.0056	0.0041	0.0069
4	8	99	64	0.0067	0.0053	0.0096
2	16	99	64	0.0035	0.0040	0.0060
4	16	99	64	0.0042	0.0049	0.0084



TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y) = (\text{REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION})$

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	DP	PA	(24, 24)
2	8	4	32	0.1862	0.1665	0.1677
4	8	4	32	0.1915	0.1751	0.1754
2	16	4	32	0.1918	0.2010	0.1987
4	16	4	32	0.1926	0.1599	0.1735
2	8	99	32	0.0091	0.0067	0.0099
4	8	99	32	0.0092	0.0066	0.0117
2	16	99	32	0.0093	0.0098	0.0134
4	16	99	32	0.0095	0.0055	0.0110
2	8	4	64	0.1892	0.1729	0.1694
4	8	4	64	0.1916	0.1728	0.1820
2	16	4	64	0.1930	0.1841	0.1810
4	16	4	64	0.1945	0.1932	0.1832
2	8	99	64	0.0090	0.0061	0.0090
4	8	99	64	0.0092	0.0070	0.0111
2	16	99	64	0.0091	0.0095	0.0125
4	16	99	64	0.0093	0.0094	0.0138

TABLE A6 REPLENISHMENT FREQUENCY

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C(OUT)	C(PIX)	DP	PA	(24,24)
		/C(IN)	/C(IN)			
2	8	4	32	0.252	0.256	0.255
4	8	4	32	0.228	0.253	0.250
2	16	4	32	0.350	0.448	0.443
4	16	4	32	0.318	0.367	0.383
2	8	99	32	0.261	0.237	0.235
4	8	99	32	0.237	0.237	0.237
2	16	99	32	0.398	0.490	0.484
4	16	99	32	0.370	0.358	0.376
2	8	4	64	0.174	0.190	0.192
4	8	4	64	0.160	0.193	0.194
2	16	4	64	0.260	0.269	0.268
4	16	4	64	0.243	0.264	0.264
2	8	99	64	0.188	0.179	0.179
4	8	99	64	0.170	0.185	0.184
2	16	99	64	0.277	0.251	0.250
4	16	99	64	0.252	0.255	0.254

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

(X, Y, Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)

COLUMN (1) \* EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST

COLUMN (2) + : BIAS FOR G.C.: POSITIVE : - : NEGATIVE : # : SIGNIFICANTLY POSITIVE : = : SIGNIFICANTLY NEGATIVE

SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

LEADTIME	MEAN	(1)		(2)	
		C(OUT) /C(IN)	C(FIX) /C(IN)	(24, 24, 24)	(24, 24, 24)
2	8	4	32	1.4	+
4	8	4	32	3.3	+
2	16	4	32	1.9	-
4	16	4	32	3.5	+
2	8	99	32	4.9	+
4	8	99	32	9.5	+
2	16	99	32	8.1	+
4	16	99	32	10.7	+
2	8	4	64	1.5	+
4	8	4	64	2.8	+
2	16	4	64	1.2	+
4	16	4	64	4.2	+
2	8	99	64	6.3	+
4	8	99	64	8.6	+
2	16	99	64	7.2	+
4	16	99	64	13.9	+

TABLE A8 ESTIMATED STANDARD DEVIATION OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y, Z) =$  (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR  
 REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)

LEADTIME	MEAN	$C(OUT) / C(IN)$	$C(FIX) / C(IN)$	(24, 24, 24)
2	8	4	32	3.7
4	8	4	32	4.9
2	16	4	32	4.0
4	16	4	32	6.2
2	8	99	32	8.9
4	8	99	32	13.0
2	16	99	32	13.4
4	16	99	32	12.8
2	8	4	64	4.6
4	8	4	64	5.5
2	16	4	64	5.0
4	16	4	64	5.4
2	8	99	64	9.5
4	8	99	64	12.0
2	16	99	64	11.3
4	16	99	64	18.5



TABLE A9 VALUES FOR LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN		C(OUT)		C(FIX)		C(IN)		C(IN)		PERIOD IN CYCLE											
										1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	(24,24)	9	9	10	9	9	9	9	9	8	7	13	19	38	47	41	25	14
					9	9	9	9	9	9	9	9	9	9	14	24	39	44	39	24	14
					8	8	7	7	7	7	7	7	7	6	7	15	24.2	44.8	39.6	24.2	14.1
8	4	64	PA	(24,24)	6	6	6	6	6	6	6	6	6	5	11	20	35	40	35	20	11
					6	6	6	6	6	6	6	6	6	6	11.1	20.6	35.4	40.4	35.4	20.6	11.1
					24	24	24	24	24	24	24	24	23	21	30	42	64	72	64	44	31
8	99	32	PA	(24,24)	24	24	24	24	24	24	24	24	24	21	31	44	64	70	64	44	31
					24	24	24	24	24	24	24	24	24	21	31	44	64	70	64	44	31
					23	23	23	23	23	23	23	23	21	21	27	40	63	71	62	43	29
8	99	64	PA	(24,24)	22	22	22	22	22	22	22	22	22	22	29	42	61	67	61	42	29
					22	22	22	22	22	22	22	22	22	22	29	42	61	67	61	42	29
					19	19	20	20	20	20	20	20	20	15	27	45	78	95	85	53	31
16	4	32	PA	(24,24)	20	20	20	20	20	20	20	20	20	20	30	50	82	92	82	50	30
					20	20	20	20	20	20	20	20	20	20	30	50	82	92	82	50	30
					17	17	18	18	18	18	18	18	16	13	23	39	74	91	82	49	27
16	4	64	PA	(24,24)	17	17	17	17	17	17	17	17	17	17	26	46	76	86	76	45	26
					16	16	16	16	16	16	16	16	16	16	26.5	46.4	76.8	87.1	76.8	46.4	26.5
					39	39	39	39	39	39	39	39	39	33	50	76	114	130	117	79	53
16	99	32	PA	(24,24)	39	39	39	39	39	39	39	39	39	39	53	78	115	127	115	78	53
					39	39	39	39	39	39	39	39	39	39	53	78	115	127	115	78	53
					40	40	40	40	40	40	40	40	40	40	53.6	79.3	128.5	128.5	116.3	79.3	53.6
16	99	64	PA	(24,24)	38	38	38	38	38	38	38	38	38	38	50	73	112	123	112	77	51
					37	37	37	37	37	37	37	37	37	37	50	75	111	123	111	75	50
					37	37	37	37	37	37	37	37	37	37	51.0	76.0	124.3	124.3	112.4	76.0	51.0

TABLE A9 VALUES FOR FIG 5

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN		C(OUT)		C(PIN)		C(IN)		PERIOD IN CYCLE											
								1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	4	32	4	32	4	25	25	24	27	33	34	51	72	69	59	42	32
								25	25	25	25	25	25	33	69	75	69	48	33
								(24,24)	25.7	25.7	25.7	25.7	33.7	48.4	69.4	76.2	69.4	46.4	33.7
8	4	64	4	64	4	64	4	31	30	32	33	34	34	77	79	74	64	55	37
								30	30	30	30	30	39	55	77	84	77	55	39
								(24,24)	30.4	30.4	30.4	30.4	39.2	55.4	77.9	85.2	77.9	55.4	39.2
8	99	32	4	32	4	32	4	39	38	36	37	54	54	53	92	87	75	57	45
								40	40	40	40	40	50	68	94	101	94	68	50
								(24,24)	40.8	40.8	40.8	40.8	51.3	69.8	95.2	103.2	95.2	69.8	51.3
8	99	64	4	64	4	64	4	46	45	46	46	57	57	77	103	92	79	61	50
								43	42	42	41	55	54	98	98	92	79	61	50
								(24,24)	46.3	46.3	46.3	46.3	57.7	77.9	104.9	113.5	104.9	77.9	57.7
16	4	32	4	32	4	32	4	45	44	41	40	44	43	67	127	130	110	76	54
								43	43	43	43	43	57	83	102	113	102	83	57
								(24,24)	42.9	42.9	42.9	42.9	57.1	84.1	104.2	114.2	104.2	84.1	57.1
16	4	64	4	64	4	64	4	51	48	46	47	67	67	100	145	137	116	84	63
								49	49	49	49	49	64	93	133	146	133	93	64
								(24,24)	49.3	49.3	49.3	49.3	64.7	93.6	134.4	147.7	134.4	93.6	64.7
16	99	32	4	32	4	32	4	62	61	57	53	67	67	90	129	155	132	95	71
								62	62	62	62	80	80	111	126	139	126	111	80
								(24,24)	62.8	62.8	62.8	62.8	80.3	112.4	129.7	140.4	129.7	112.4	80.3
16	99	64	4	64	4	64	4	70	66	61	55	67	67	94	133	162	138	102	81
								69	69	69	69	88	88	122	168	182	168	122	88
								(24,24)	70.3	70.3	70.3	70.3	89.1	123.2	170.0	185.0	170.0	123.2	89.1

TABLE A9 VALUES FOR BIC S - LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN C(OUT) C(IX)		C(IN) /C(IN)		1	2	3	4	5	PERIOD IN CYCLE				8	9	10	11	12
8	4	32	DP	17	15	15	19	25	21	32	34	22	34	22	18	17	18
				PA	16	16	16	16	19	24	30	31	30	31	30	24	19
8	4	64	(24,24)	23	23	25	26	23	27	62	29.8	29	43	29	26	24.3	19.6
				DP	24	24	24	24	28	35	42	44	42	44	42	35	25
8	99	32	(24,24)	23.8	23.8	23.8	23.8	23.8	28.0	34.7	42.5	44.7	42.5	44.7	42.5	34.7	28.0
				DP	15	14	12	14	24	11	28	15	28	15	11	13	14
8	99	64	(24,24)	16	16	16	16	16	19	24	30	31	30	31	30	24	19
				PA	16.6	16.6	16.6	16.6	19.6	24.3	29.8	31.4	29.8	31.4	29.8	24.3	19.6
8	99	64	(24,24)	20	19	19	20	34	27	42	52	44	42	44	42	35	28
				DP	24	24	24	24	28	34.7	42.5	44.7	42.5	44.7	42.5	34.7	28.0
16	4	32	(24,24)	26	24	20	20	23	40	22	49	35	25	21	20	23	23
				PA	23	23	23	23	27	33	20	21	20	21	20	33	27
16	4	64	(24,24)	22.7	22.7	22.7	22.7	22.7	26.7	33.1	41.7	43.1	21.8	21.4	21.8	33.1	26.7
				DP	34	30	31	31	44	61	71	46	34	46	34	35	36
16	99	32	(24,24)	32	32	32	32	32	38	47	57	60	57	60	57	47	38
				PA	32.4	32.4	32.4	32.4	39.1	47.2	57.6	60.6	57.6	60.6	57.6	47.2	38.1
16	99	64	(24,24)	23	22	17	14	23	44	14	15	25	15	25	15	16	18
				DP	23	23	23	23	27	33	11	12	11	12	11	33	27
16	99	32	(24,24)	22.7	22.7	22.7	22.7	22.7	26.7	33.1	41.7	43.1	13.4	11.9	13.4	33.1	26.7
				PA	32	28	17	17	44	60	61	34	23	34	23	25	30
16	99	64	(24,24)	32	32	32	32	32	38	47	57	60	57	60	57	47	38
				DP	32.4	32.4	32.4	32.4	38.1	47.2	57.6	60.6	57.6	60.6	57.6	47.2	38.1

TABLE A9 VALUES FOR LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III  
 REPLENISHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	1	2	3	4	5	6	7	8	9	10	11	12
	/C(IN)	/C(IN)												
8	4	32	DP 18	16	15	20	28	46	62	68	61	49	33	23
			PA 18	18	18	23	33	49	59	65	59	49	33	23
			(24,24) 18.1	18.1	18.1	23.2	33.8	49.6	60.3	65.6	60.3	49.6	33.8	23.2
8	4	64	DP 15	15	15	16	22	44	60	66	59	47	31	20
			PA 15	15	15	20	30	45	55	60	55	45	30	20
			(24,24) 15.2	15.2	15.2	20.1	30.2	45.5	55.8	61.0	55.8	45.5	30.2	20.1
8	99	32	DP 35	35	33	40	52	73	90	96	88	74	55	42
			PA 35	35	35	42	56	75	88	94	83	75	56	42
			(24,24) 36.1	36.1	36.1	43.2	56.8	76.5	89.2	95.6	89.2	76.5	56.8	43.2
8	59	64	DP 33	32	32	39	50	72	89	94	86	72	53	40
			PA 33	33	33	40	53	72	84	90	84	72	53	40
			(24,24) 34.0	34.0	34.0	40.9	54.1	73.2	85.7	91.9	85.7	73.2	54.1	40.9
16	4	32	DP 39	37	33	43	62	94	122	138	125	102	69	48
			PA 37	37	37	47	58	100	121	132	121	100	68	47
			(24,24) 37.5	37.5	37.5	48.0	61.1	101.0	122.4	133.0	122.4	101.0	69.1	48.0
16	4	64	DP 35	33	29	38	57	88	119	134	122	98	66	45
			PA 33	33	33	43	54	95	115	126	115	95	64	43
			(24,24) 33.7	33.7	33.7	43.9	54.4	95.6	116.5	127.0	116.5	95.6	64.4	43.9
16	99	32	DP 61	59	59	70	95	133	160	175	162	136	99	73
			PA 61	61	61	74	99	135	159	171	159	135	99	74
			(24,24) 61.5	61.5	61.5	74.4	99.7	136.6	160.8	172.7	160.8	136.6	99.7	74.4
16	99	64	DP 59	58	56	68	93	129	159	173	160	134	97	72
			PA 58	58	58	70	95	131	154	166	154	131	95	70
			(24,24) 58.6	58.6	58.6	71.3	96.0	132.2	156.0	167.8	156.0	132.2	96.0	71.3



TABLE A9 VALUES FOR BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III  
 REPLENISHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	PERIOD IN CYCLE												
			1	2	3	4	5	6	7	8	9	10	11	12	
8	4	32	DP	35	37	41	43	59	86	87	86	80	68	52	41
			PA	35	35	35	42	55	75	68	95	88	75	55	42
			(24,24)	35.2	35.2	35.2	42.4	56.2	76.1	89.0	95.4	89.0	76.1	56.2	42.4
			DP	42	42	43	43	91	91	91	91	84	73	57	47
8	4	64	PA	40	40	40	47	52	83	96	102	96	83	62	47
			(24,24)	39.8	39.8	39.8	47.5	52.3	83.3	96.8	103.5	96.8	83.3	62.3	47.5
			DP	50	50	50	64	94	110	110	108	100	86	56	41
			PA	52	52	52	61	78	101	117	124	117	101	78	61
8	32	32	(24,24)	53.3	53.3	53.3	62.3	79.2	103.0	118.0	125.4	118.0	103.0	79.2	62.3
			DP	53	66	65	65	115	115	113	111	103	90	72	59
			PA	58	58	58	67	85	110	125	132	125	110	85	67
			(24,24)	58.7	58.7	58.7	68.3	86.1	111.0	126.8	134.5	126.8	111.0	86.1	68.3
16	4	32	DP	60	61	62	83	116	138	164	164	151	127	73	
			PA	60	60	60	73	98	136	160	172	160	136	98	73
			(24,24)	60.9	60.9	60.9	74.1	99.7	137.0	159.0	165.4	159.0	137.0	99.7	74.1
			DP	66	65	65	83	116	170	171	169	157	134	101	73
16	4	64	PA	66	66	66	80	107	146	170	184	170	146	107	80
			(24,24)	67.3	67.3	67.3	81.1	108.0	146.8	172.2	184.7	172.2	146.8	108.0	81.1
			DP	79	75	87	113	110	149	196	194	179	152	116	93
			PA	84	84	84	100	129	171	198	211	198	171	129	100
16	32	32	(24,24)	84.9	84.9	84.9	100.5	130.3	172.5	196.4	201.6	196.4	172.5	130.3	100.5
			DP	84	88	88	114	152	204	202	198	184	158	123	99
			PA	91	91	91	107	138	182	209	224	209	182	138	107
			(24,24)	92.2	92.2	92.2	108.5	139.6	183.5	211.8	225.6	211.8	183.5	139.6	108.5

TABLE A9 VALUES FOR BIG S - LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III

REFRESHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X, Y) : Y = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	PERIOD IN CYCLE											
			1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	17	21	26	23	31	40	25	18	19	19	18
			PA	17	17	17	19	22	26	29	30	26	22	19
8	4	64	(24, 24)	17.2	17.2	17.2	19.1	22.4	26.5	28.7	29.8	26.5	22.4	19.1
			DP	27	27	28	27	69	47	31	25	26	26	27
8	99	32	PA	25	25	25	27	32	38	41	42	38	32	27
			(24, 24)	24.7	24.7	24.7	27.4	32.1	37.8	41.1	42.6	41.1	32.8	27.4
8	99	64	DP	15	15	17	24	32	37	20	12	12	13	14
			PA	17	17	17	19	22	26	29	30	26	22	19
8	99	32	(24, 24)	17.2	17.2	17.2	19.1	22.4	26.5	28.7	29.8	26.5	22.4	19.1
			DP	20	34	34	26	65	43	24	17	18	19	19
8	99	64	PA	25	25	25	27	32	38	41	42	38	32	27
			(24, 24)	24.7	24.7	24.7	27.4	32.1	37.8	41.1	42.6	41.1	32.8	27.4
16	4	32	DP	22	24	29	40	54	44	42	26	25	26	25
			PA	23	23	23	26	30	36	39	40	36	30	26
16	4	64	(24, 24)	23.5	23.5	23.5	26.1	30.6	35.9	36.6	32.4	35.9	30.6	26.1
			DP	31	32	54	45	59	82	52	35	36	35	33
16	99	32	PA	33	33	33	37	43	51	55	58	51	43	37
			(24, 24)	33.6	33.6	33.6	37.3	43.6	51.3	55.7	57.8	51.3	43.6	37.3
16	99	64	DP	18	16	28	43	15	16	36	19	16	17	20
			PA	23	23	23	26	30	36	39	40	36	30	26
16	99	32	(24, 24)	23.5	23.5	23.5	26.1	30.6	35.9	35.6	28.9	35.9	30.6	26.1
			DP	25	33	32	46	59	75	43	25	24	26	27
16	99	64	PA	33	33	33	37	43	51	55	58	51	43	37
			(24, 24)	33.6	33.6	33.6	37.3	43.6	51.3	55.7	57.8	51.3	43.6	37.3

TABLE A10 STANDARD DEVIATION OF LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	32	1.5	1.5	1.5	1.5	1.5	2.2	3.5	5.6	6.3	5.6	3.5	2.2
4	8	4	32	2.5	2.5	2.5	3.2	4.5	6.5	7.9	8.6	7.9	6.5	4.5	3.2
2	8	4	64	1.3	1.3	1.3	1.3	1.3	2.0	3.3	5.3	5.9	5.3	3.3	2.0
4	8	4	64	2.3	2.3	2.3	2.9	4.2	6.2	7.5	8.2	7.5	6.2	4.2	2.9
2	8	99	32	4.4	4.4	4.4	4.4	4.4	5.4	7.3	9.9	10.7	9.9	7.3	5.4
4	8	99	32	5.7	5.7	5.7	6.6	8.3	10.7	12.3	13.1	12.3	10.7	8.3	6.6
2	8	99	64	4.0	4.0	4.0	4.0	4.0	4.9	6.7	9.2	10.0	9.2	6.7	4.9
4	8	99	64	5.2	5.2	5.2	6.0	7.7	10.0	11.5	12.3	11.5	10.0	7.7	6.0
2	16	4	32	2.1	2.1	2.1	2.1	2.1	3.1	5.0	7.9	8.8	7.9	5.0	3.1
4	16	4	32	3.5	3.5	3.5	4.5	6.4	9.2	11.1	12.0	11.1	9.2	6.4	4.5
2	16	4	64	1.9	1.9	1.9	1.9	1.9	2.8	4.7	7.5	8.5	7.5	4.7	2.8
4	16	4	64	3.4	3.4	3.4	4.2	6.1	8.9	10.7	11.7	10.7	8.9	6.1	4.2
2	16	99	32	5.3	5.3	5.3	5.3	5.3	6.6	8.9	12.3	13.4	12.3	8.9	6.6
4	16	99	32	7.1	7.1	7.1	8.2	10.4	13.6	15.6	16.7	15.6	13.6	10.4	8.2
2	16	99	64	4.8	4.8	4.8	4.8	4.8	6.0	8.3	11.5	12.5	11.5	8.3	6.0
4	16	99	64	6.5	6.5	6.5	7.6	9.8	12.8	14.8	15.9	14.8	12.8	9.8	7.6

TABLE A10 STANDARD DEVIATION OF BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL III  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (L,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

RADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	PERIOD IN CYCLE						7	8	9	10	11	12
				1	2	3	4	5	6						
2	8	4	32	2.6	2.6	2.6	2.6	2.6	3.5	5.1	7.5	8.3	7.5	5.1	3.5
4	8	4	32	3.6	3.6	3.6	4.4	6.0	8.2	9.7	10.4	9.7	8.2	6.0	4.4
2	8	4	64	2.7	2.7	2.7	2.7	2.7	3.6	5.4	7.9	8.7	7.9	5.4	3.6
4	8	4	64	3.8	3.8	3.8	4.6	6.2	8.5	10.0	10.7	10.0	8.5	6.2	4.6
2	8	99	32	5.4	5.4	5.4	5.4	5.4	6.7	8.8	11.8	12.7	11.8	8.8	6.7
4	8	99	32	6.7	6.7	6.7	7.8	9.6	12.3	14.1	15.0	14.1	12.3	9.6	7.8
2	8	99	64	5.4	5.4	5.4	5.4	5.4	6.6	8.9	11.9	12.8	11.9	8.9	6.6
4	8	99	64	6.7	6.7	6.7	7.7	9.7	12.3	14.1	14.9	14.1	12.3	9.7	7.7
2	16	4	32	3.2	3.2	3.2	3.2	3.2	4.3	6.5	8.5	9.9	8.5	6.5	4.3
4	16	4	32	4.6	4.6	4.6	5.7	7.8	10.7	10.7	10.4	10.7	10.7	7.8	5.7
2	16	4	64	3.3	3.3	3.3	3.3	3.3	4.5	6.7	10.0	11.1	10.0	6.7	4.5
4	16	4	64	4.9	4.9	4.9	5.9	8.0	11.1	13.1	14.1	13.1	11.1	8.0	5.9
2	16	99	32	6.3	6.3	6.3	6.3	6.3	7.9	10.5	13.0	13.4	13.0	10.5	7.9
4	16	99	32	8.1	8.1	8.1	9.4	11.9	15.4	16.2	17.0	16.2	15.4	11.9	9.4
2	16	99	64	6.3	6.3	6.3	6.3	6.3	7.8	10.5	14.2	15.4	14.2	10.5	7.8
4	16	99	64	8.0	8.0	8.0	9.3	11.8	15.2	17.4	18.6	17.4	15.2	11.8	9.3



Appendix E  
Single-item Data

Model IV

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

	<u>page</u>
Table A1 Average Total Cost	E1
A3 Period-End Inventory	E2
A4 Period-End Backlog as Proportion of Mean Demand	E3
A5 Frequency of Periods with Backlog	E4
A6 Replenishment Frequency	E5
A7 Estimated Bias of Forecast of Total Cost	E6
A8 Estimated Standard Deviation of Forecast of Total Cost	E7
A9 Values for ( $s_i, S_i$ )	E8 to E13
A10 Standard Deviations of ( $s_i, S_i$ ) Values	E14 to E15

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

TABLE A1 AVERAGE TOTAL COST

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL DP ARE % EXCESS OVER DP VALUE

LEADTIME	MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	DP	PA	(24, 24)
2	8	4	32	25.0	1.5	5.6
4	8	4	32	27.8	2.2	9.4
2	16	4	32	34.9	0.9	4.1
4	16	4	32	38.8	1.3	7.2
2	8	99	32	41.1	2.7	13.8
4	8	99	32	47.2	2.9	17.1
2	16	99	32	55.8	2.4	11.6
4	16	99	32	64.2	3.0	16.0
2	8	4	64	32.0	2.2	5.3
4	8	4	64	34.4	2.9	8.0
2	16	4	64	45.0	1.7	4.0
4	16	4	64	48.4	2.4	7.7
2	8	99	64	48.6	3.7	11.5
4	8	99	64	54.3	4.0	15.2
2	16	99	64	66.9	3.0	9.9
4	16	99	64	74.8	3.2	12.7

TABLE A3 PERIOD-END INVENTORY

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	DP	PA	(24,24)
2	8	4	32	11.3	11.6	12.6
4	8	4	32	13.5	13.9	15.4
2	16	4	32	15.7	16.5	16.9
4	16	4	32	18.6	19.1	20.9
2	8	99	32	27.8	30.4	32.6
4	8	99	32	33.2	36.4	39.6
2	16	99	32	37.9	40.6	42.1
4	16	99	32	45.7	48.7	51.7
2	8	4	64	13.9	14.0	14.9
4	8	4	64	15.8	15.4	17.0
2	16	4	64	19.6	19.7	21.1
4	16	4	64	22.4	22.6	24.7
2	8	99	64	30.9	33.8	35.9
4	8	99	64	36.3	39.0	41.9
2	16	99	64	42.1	45.8	48.2
4	16	99	64	49.0	53.2	56.2

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C(OUT)	C(FIX)	DP	PA	(24,24)
		C(IN)	/C(IN)			
2	8	4	32	0.1736	0.1628	0.1633
4	8	4	32	0.2071	0.1892	0.2028
2	16	4	32	0.1133	0.1088	0.1136
4	16	4	32	0.1390	0.1361	0.1438
2	8	99	32	0.0059	0.0044	0.0074
4	8	99	32	0.0066	0.0051	0.0096
2	16	99	32	0.0036	0.0033	0.0052
4	16	99	32	0.0043	0.0040	0.0074
2	8	4	64	0.1824	0.1881	0.1892
4	8	4	64	0.2206	0.2279	0.2304
2	16	4	64	0.1260	0.1209	0.1159
4	16	4	64	0.1539	0.1379	0.1433
2	8	99	64	0.0057	0.0054	0.0076
4	8	99	64	0.0068	0.0065	0.0106
2	16	99	64	0.0037	0.0032	0.0047
4	16	99	64	0.0044	0.0041	0.0067



TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C(OUT)	C(FIX)	DP	PA	(24,24)
		C(IN)	/C(IN)			
2	8	4	32	0.1912	0.1773	0.1718
4	8	4	32	0.1905	0.1746	0.1757
2	16	4	32	0.1895	0.1795	0.1806
4	16	4	32	0.1930	0.1848	0.1840
2	8	99	32	0.0093	0.0068	0.0103
4	8	99	32	0.0089	0.0068	0.0112
2	16	99	32	0.0091	0.0082	0.0114
4	16	99	32	0.0093	0.0083	0.0133
2	8	4	64	0.1882	0.1856	0.1829
4	8	4	64	0.1913	0.1916	0.1856
2	16	4	64	0.1916	0.1812	0.1709
4	16	4	64	0.1954	0.1768	0.1733
2	8	99	64	0.0089	0.0080	0.0104
4	8	99	64	0.0092	0.0083	0.0121
2	16	99	64	0.0092	0.0077	0.0103
4	16	99	64	0.0094	0.0083	0.0120

TABLE A6 REPLENISHMENT FREQUENCY

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	DP	PA	(24, 24)
2	8	4	32	0.255	0.268	0.269
4	8	4	32	0.240	0.266	0.267
2	16	4	32	0.372	0.367	0.381
4	16	4	32	0.351	0.358	0.359
2	8	99	32	0.272	0.261	0.260
4	8	99	32	0.276	0.255	0.254
2	16	99	32	0.385	0.351	0.373
4	16	99	32	0.365	0.348	0.347
2	8	4	64	0.192	0.199	0.200
4	8	4	64	0.180	0.199	0.200
2	16	4	64	0.271	0.285	0.285
4	16	4	64	0.253	0.284	0.284
2	8	99	64	0.207	0.193	0.192
4	8	99	64	0.197	0.193	0.192
2	16	99	64	0.298	0.281	0.278
4	16	99	64	0.294	0.273	0.273

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)

COLUMN (1) &amp; EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST

COLUMN (2) + : BIAS FOR O.C.: POSITIVE : - : NEGATIVE : # : SIGNIFICANTLY POSITIVE : = : SIGNIFICANTLY NEGATIVE

SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

		(1)		(2)	
		(24, 24, 24)		(24, 24, 24)	
LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)		
2	8	4	32	#	#
4	8	4	32	#	#
2	16	4	32	#	#
4	16	4	32	#	#
2	8	99	32	#	#
4	8	99	32	#	#
2	16	99	32	#	#
4	16	99	32	#	#
2	8	4	64	#	#
4	8	4	64	#	#
2	16	4	64	#	#
4	16	4	64	#	#
2	8	99	64	#	#
4	8	99	64	#	#
2	16	99	64	#	#
4	16	99	64	#	#

TABLE A8 ESTIMATED STANDARD DEVIATION OF FORECAST OF TOTAL COST

## NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 $(X, Y, Z)$  = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR  
 REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)

LEADTIME	MEAN	C (OUT) /C(IN)	C (FIX) /C(IN)	(24, 24, 24)
2	8	4	32	3.6
4	8	4	32	5.2
2	16	4	32	4.3
4	16	4	32	6.2
2	8	99	32	7.5
4	8	99	32	10.2
2	16	99	32	11.2
4	16	99	32	11.9
2	8	4	64	4.2
4	8	4	64	5.1
2	16	4	64	4.4
4	16	4	64	6.2
2	8	99	64	9.6
4	8	99	64	11.6
2	16	99	64	14.7
4	16	99	64	13.2



TABLE A9 VALUES FOR LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN		C(OUT)		C(FIX)		PERIOD IN CYCLE											
		/C(IN)		/C(IN)		1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	(24,24)	11.4	11	6	5	9	15	21	27	33	34	31	25	18
						11	7	7	11	17	23	29	33	33	29	23	17
						9	4	3	7	13	17	24	30	32	29	23	16
8	4	64	PA	(24,24)	8.6	8	5	5	8	14	20	25	29	29	25	20	14
						26	5.1	5.1	8.6	14.2	19.9	25.8	29.8	29.8	25.8	19.9	14.2
						27	20	19	25	34	42	50	57	58	52	44	35
8	99	32	PA	(24,24)	27.6	27	21	21	27	35	43	51	57	57	51	43	35
						25	18	18	24	33	40	49	55	56	51	43	34
						25	19	19	25	33	41	49	54	54	49	41	33
8	99	64	PA	(24,24)	25.8	25	20	20	25.8	34.2	42.2	50.0	55.2	55.2	50.0	42.2	34.2
						25	16	14	22	34	46	59	69	70	63	52	39
						24	16	16	24	37	49	61	70	70	61	49	37
16	4	32	PA	(24,24)	24.8	24	16	16.7	24.8	37.1	49.6	62.0	70.5	70.5	62.0	49.6	37.1
						23	12	10	19	30	41	54	64	66	60	49	36
						21	13	13	21	33	44	56	65	65	56	44	33
16	4	64	PA	(24,24)	21.2	21	13	13.6	21.2	33.0	45.0	57.1	65.3	65.3	57.1	45.0	33.0
						46	33	33	44	50	75	90	101	102	93	78	62
						45	35	35	45	61	76	91	101	101	91	76	61
16	99	32	PA	(24,24)	46.4	44	32	31.5	46.4	57.3	77.6	92.7	102.6	102.6	92.7	77.6	62.3
						44	32	31	42	57	73	88	99	100	91	76	60
						43	33	33	43	58	73	88	97	97	88	73	58
16	99	64	PA	(24,24)	44.0	44	33.4	33.4	44.0	59.4	74.4	89.1	98.9	98.9	89.1	74.4	59.4

TABLE A9 VALUES FOR BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN		C (OUT) / C (IN)		C (FIX) / C (IN)		PERIOD IN CYCLE											
		1	2	3	4	5	6	7	8	9	10	11	12				
8	DP	27	25	27	34	42	50	59	63	59	51	41	33				
	PA	29	22	22	29	38	47	55	61	61	55	47	38				
	(24,24)	29.4	22.7	22.7	29.4	38.6	47.5	56.0	61.7	61.7	56.0	47.5	38.6				
8	DP	33	32	35	41	49	61	71	70	65	56	46	37				
	PA	34	27	27	34	44	54	62	69	69	62	54	44				
	(24,24)	34.4	27.1	27.1	34.4	44.6	54.3	63.5	69.6	69.6	63.5	54.3	44.6				
8	DP	39	36	42	51	60	66	80	81	76	67	56	46				
	PA	45	36	36	45	56	67	77	85	85	77	67	56				
	(24,24)	45.6	36.9	36.9	45.6	57.6	68.6	79.0	85.8	85.8	79.0	68.6	57.6				
8	DP	43	44	53	56	67	86	91	88	81	70	59	50				
	PA	51	41	41	51	63	75	86	94	94	86	75	63				
	(24,24)	51.6	42.1	42.1	51.6	64.7	76.6	87.8	95.0	95.0	87.8	76.6	64.7				
16	DP	46	39	38	50	67	84	99	107	106	94	76	59				
	PA	48	37	37	48	66	82	97	108	108	97	82	66				
	(24,24)	49.5	37.7	37.7	49.5	66.1	82.3	97.0	102.4	102.4	97.0	82.3	66.1				
16	DP	51	48	51	66	83	99	112	121	118	101	81	64				
	PA	56	43	43	56	74	90	107	119	119	107	90	74				
	(24,24)	56.4	43.7	43.7	56.4	74.4	91.6	108.4	119.3	119.3	108.4	91.6	74.4				
16	DP	61	55	56	73	93	113	128	136	131	115	95	76				
	PA	69	56	56	69	90	109	127	139	139	127	109	90				
	(24,24)	71.1	56.5	56.5	71.1	91.3	110.4	127.3	132.0	132.0	127.3	110.4	91.3				
16	DP	68	62	75	94	115	136	151	151	141	122	100	81				
	PA	78	63	63	78	99	119	139	151	151	139	119	99				
	(24,24)	79.2	63.5	63.5	79.2	100.8	121.1	140.4	152.9	152.9	140.4	121.1	100.8				

TABLE A9 VALUES FOR BIG S - LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV  
 REPLENISHMENT LEADTIME = 2

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C (OUT)	C (PIX)	1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	16	19	22	25	27	29	32	30	25	20	16
			PA	18	15	15	18	21	24	26	28	28	26	24
			(24,24)	16.0	15.3	15.3	18.0	21.3	24.0	26.4	27.9	27.9	26.4	24.0
8	4	64	DP	24	28	32	34	36	44	47	40	33	27	23
			PA	26	22	22	26	30	34	37	40	40	37	34
			(24,24)	25.8	22.0	22.0	25.8	30.5	34.4	37.8	39.8	39.8	37.8	34.4
8	99	32	DP	13	16	23	26	26	24	30	24	18	15	12
			PA	18	15	15	18	21	24	26	28	28	26	24
			(24,24)	18.0	15.3	15.3	18.0	21.3	24.0	26.4	27.9	27.9	26.4	24.0
8	99	64	DP	18	26	35	32	34	46	42	33	25	19	16
			PA	26	22	22	26	30	34	37	40	40	37	34
			(24,24)	25.8	22.0	22.0	25.8	30.5	34.4	37.8	39.8	39.8	37.8	34.4
16	4	32	DP	21	23	24	28	31	36	40	38	36	31	24
			PA	24	21	21	24	29	33	36	38	38	36	33
			(24,24)	24.7	21.0	21.0	24.7	29.1	32.8	35.0	31.9	31.9	35.0	32.8
16	4	64	DP	28	36	41	47	51	58	58	57	52	41	32
			PA	35	30	30	35	41	46	51	54	54	51	46
			(24,24)	35.2	30.1	30.1	35.2	41.5	46.7	51.3	54.0	54.0	51.3	46.7
16	99	32	DP	15	22	23	29	33	38	38	35	29	22	17
			PA	24	21	21	24	29	33	36	38	38	36	33
			(24,24)	24.7	21.0	21.0	24.7	29.1	32.8	34.6	29.4	29.4	34.6	32.8
16	99	64	DP	24	30	44	52	52	42	48	52	41	31	24
			PA	35	30	30	35	41	46	51	54	54	51	46
			(24,24)	35.2	30.1	30.1	35.2	41.5	46.7	51.3	54.0	54.0	51.3	46.7

TABLE A9 VALUES FOR LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV  
 REPLEISHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	C(IN)	PERIOD IN CYCLE											
				1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	18	17	21	29	39	49	56	57	53	43	32	23
			PA	19	19	24	32	42	51	55	55	51	42	32	24
			(24,24)	20.0	20.0	24.2	32.6	43.3	51.7	56.0	56.0	56.0	43.3	32.6	24.2
8	4	64	DP	15	15	19	24	35	46	53	55	50	41	30	20
			PA	17	17	20	28	38	46	51	51	46	38	28	20
			(24,24)	17.0	17.0	20.9	29.1	39.3	47.5	51.7	51.7	47.5	39.3	29.1	20.9
8	99	32	DP	37	36	41	52	65	76	83	84	78	67	54	43
			PA	38	38	43	54	67	77	82	82	77	67	54	43
			(24,24)	38.9	38.9	44.5	55.4	68.7	79.1	84.3	84.3	79.1	68.7	55.4	44.5
8	99	64	DP	35	34	40	49	53	75	82	82	76	65	52	41
			PA	36	36	41	51	64	74	79	79	74	64	51	41
			(24,24)	36.6	36.6	42.1	52.7	65.7	75.8	80.8	80.8	75.8	65.7	52.7	42.1
16	4	32	DP	39	37	45	62	83	101	111	115	107	90	67	49
			PA	41	41	49	66	87	104	112	112	104	87	66	49
			(24,24)	41.3	41.3	49.8	66.9	88.2	105.3	113.8	113.8	105.3	88.2	66.9	49.8
16	4	64	DP	34	34	40	56	78	95	107	111	104	86	64	45
			PA	37	37	45	61	82	98	107	107	98	82	61	45
			(24,24)	37.4	37.4	45.7	62.1	83.0	99.7	108.1	108.1	99.7	83.0	62.1	45.7
16	99	32	DP	64	63	73	93	118	138	149	151	142	121	96	75
			PA	65	65	75	96	120	139	149	149	139	120	96	75
			(24,24)	66.5	66.5	76.9	97.2	122.1	141.7	151.4	151.4	141.7	122.1	97.2	76.9
16	99	64	DP	62	60	71	90	115	135	147	150	140	119	94	73
			PA	62	62	72	92	116	135	144	144	135	116	92	72
			(24,24)	63.4	63.4	73.6	93.5	118.0	137.2	146.8	146.8	137.2	118.0	93.5	73.6





TABLE A9 VALUES FOR BIG S - LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV  
 REPLENISHMENT LEADTIME = 4

STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN		C(OUT)		C(FIX)		PERIOD IN CYCLE											
		/C(IN)		/C(IN)		1	2	3	4	5	6	7	8	9	10	11	12
8	4	32	DP	22	24	28	31	33	34	27	22	18	18	20			
			PA	18	18	19	22	25	27	28	28	27	25	19			
			(24,24)	17.9	17.9	19.4	22.1	24.9	27.0	27.9	27.9	27.0	24.9	22.1			
8	4	64	DP	34	35	36	41	53	44	35	28	25	24	30			
			PA	26	26	28	31	35	38	39	39	38	35	28			
			(24,24)	25.7	25.7	27.9	31.6	35.6	38.5	39.8	39.8	38.5	35.6	31.6			
8	99	32	DP	19	25	20	23	26	30	22	15	13	12	16			
			PA	18	18	19	22	25	27	26	28	27	25	19			
			(24,24)	17.9	17.9	19.4	22.1	24.9	27.0	27.9	27.9	27.0	24.9	22.1			
8	99	64	DP	31	30	35	38	49	37	27	21	18	17	24			
			PA	26	26	28	31	35	38	39	39	38	35	28			
			(24,24)	25.7	25.7	27.9	31.6	35.6	38.5	39.8	39.8	38.5	35.6	31.6			
16	4	32	DP	29	29	33	38	41	40	42	33	26	22	26			
			PA	24	24	26	30	34	36	38	38	36	34	26			
			(24,24)	24.5	24.5	26.5	30.2	34.0	36.6	37.4	37.4	36.6	34.0	30.2			
16	4	64	DP	43	47	57	63	59	65	57	43	35	31	38			
			PA	35	35	38	43	48	52	54	54	52	48	38			
			(24,24)	35.0	35.0	37.9	43.0	48.4	52.3	54.1	54.1	52.3	48.4	43.0			
16	99	32	DP	22	27	34	40	39	35	35	25	18	16	21			
			PA	24	24	26	30	34	36	38	38	36	34	26			
			(24,24)	24.5	24.5	26.5	30.1	34.0	36.6	37.3	37.3	36.6	34.0	30.1			
16	99	64	DP	37	52	37	43	44	60	46	32	25	22	30			
			PA	35	35	38	43	48	52	54	54	52	48	38			
			(24,24)	35.0	35.0	37.9	43.0	48.4	52.3	54.1	54.1	52.3	48.4	43.0			

TABLE A10 STANDARD DEVIATION OF LITTLE S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

LEADTIME	MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	32	1.7	1.2	1.2	1.7	2.5	3.3	4.2	4.7	4.7	4.2	3.3	2.5
4	8	4	32	2.8	2.8	3.4	4.5	5.9	6.9	7.5	7.5	6.9	5.9	4.5	3.4
2	8	4	64	1.5	1.0	1.0	1.5	2.3	3.1	3.8	4.3	4.3	3.8	3.1	2.3
4	8	4	64	2.5	2.5	3.1	4.2	5.5	6.6	7.1	7.1	6.6	5.5	4.2	3.1
2	8	99	32	4.7	3.9	3.9	4.7	5.9	7.0	8.0	8.7	8.7	8.0	7.0	5.9
4	8	99	32	6.2	6.2	6.8	8.3	9.9	11.2	11.8	11.8	11.2	9.9	8.3	6.8
2	8	99	64	4.2	3.5	3.5	4.2	5.4	6.4	7.4	8.0	8.0	7.4	6.4	5.4
4	8	99	64	5.6	5.6	6.3	7.6	9.2	10.5	11.1	11.1	10.5	9.2	7.6	6.3
2	16	4	32	2.5	1.8	1.8	2.5	3.6	4.8	5.9	6.7	6.7	5.9	4.8	3.6
4	16	4	32	4.0	4.0	4.7	6.3	8.2	9.7	10.5	10.5	9.7	8.2	6.3	4.7
2	16	4	64	2.3	1.6	1.6	2.3	3.4	4.5	5.6	6.4	6.4	5.6	4.5	3.4
4	16	4	64	3.7	3.7	4.5	6.0	7.9	9.4	10.1	10.1	9.4	7.9	6.0	4.5
2	16	99	32	5.8	4.7	4.7	5.8	7.2	8.7	10.0	10.9	10.9	10.0	8.7	7.2
4	16	99	32	7.6	7.6	8.6	10.4	12.5	14.3	15.1	15.1	14.3	12.5	10.4	8.6
2	16	99	64	5.3	4.3	4.3	5.3	6.7	8.0	9.4	10.2	10.2	9.4	8.0	6.7
4	16	99	64	7.0	7.0	7.9	9.7	11.8	13.4	14.3	14.3	13.4	11.8	9.7	7.9

TABLE A10 STANDARD DEVIATION OF BIG S

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND, MODEL IV  
 STATISTICAL POLICIES COMPUTED USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES  
 (X,Y) : X = REVISION INTERVAL : Y = # PERIODS DATA USED TO REVISE PARAMETERS

LEADTIME	MEAN	C(OUT) /C(IN)	C(PIN) /C(IN)	PERIOD IN CYCLE						11	12
				1	2	3	4	5	6		
2	8	4	32	2.9	2.2	2.2	2.9	3.9	4.9	5.8	3.9
4	8	4	32	4.0	4.0	4.6	5.9	7.5	8.6	9.2	4.6
2	8	4	64	3.1	2.4	2.4	3.1	4.2	5.2	6.2	4.2
4	8	4	64	4.1	4.1	4.8	6.1	7.7	8.9	9.5	4.8
2	8	99	32	5.8	4.8	4.8	5.8	7.3	8.5	9.7	7.3
4	8	99	32	7.2	7.2	8.0	9.6	11.5	12.9	13.6	8.0
2	8	99	64	5.9	4.8	4.8	5.9	7.3	8.6	9.9	7.3
4	8	99	64	7.2	7.2	8.0	9.6	11.5	12.9	13.6	8.0
2	16	4	32	3.7	2.8	2.8	3.7	5.0	6.3	6.9	5.0
4	16	4	32	5.1	5.1	6.0	7.6	9.7	11.2	11.6	6.0
2	16	4	64	3.8	2.9	2.9	3.8	5.2	6.5	7.8	5.2
4	16	4	64	5.2	5.2	6.1	7.9	10.0	11.6	12.4	6.1
2	16	99	32	7.0	5.7	5.7	7.0	8.6	10.2	11.8	8.6
4	16	99	32	8.8	8.8	9.8	11.8	14.1	16.0	16.5	9.8
2	16	99	64	6.9	5.6	5.6	6.9	8.6	10.2	11.8	8.6
4	16	99	64	8.7	8.7	9.6	11.7	14.1	15.9	16.8	9.6



## Appendix F

### Multi-item Data for the Power Approximation

#### Stationary Model

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Optimally with Full Information.

	<u>page</u>
Table F1 Sources of Expected Total Cost	F1
F3 Sources of Aggregate Period-End Inventory	F2
F4 Sources of Aggregate Backlog Cost	F3
F5 Sources of Aggregate Replenishment Cost	F4
F6 Backlog Frequency	F5
F7 Weighted Proportion of Demand Backlogged	F6
F8 Replenishment Frequency	F7

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.

Table F1  
SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

## FULL INFORMATION, OPTIMAL CONTROL(DP)

OVERALL AGGREGATE FOR SYSTEM= 759.7			SOURCES OF TOTAL COST		
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN
MEAN	4	99	32 64	2 4	8 16
	38.6	61.4	45.2 54.8	47.4 52.6	41.8 58.2
LEADTIME	8	16.0 25.8	18.9 22.9	19.8 22.0	
	16	22.6 35.6	26.3 31.9	27.6 30.6	
C (FIX)/C (IN)	2	18.5 28.9	21.2 26.1		
	4	20.1 32.5	24.0 28.7		
LEADTIME C (FIX)/C (IN)	32	17.0 28.2			
	64	21.6 33.2			
LEADTIME C (FIX)/C (IN)	2	8.1 13.2			
	2	10.4 15.7			
	4	8.9 15.0			
LEADTIME C (FIX)/C (IN)	4	11.2 17.5			

Table F3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		439.3	PERCENT OF TOTAL COST= 57.8		MEAN
		C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	
MEAN	4	99	32 64	2 4	8 16
	30.0	70.0	46.9 53.1	46.0 54.0	42.1 57.9
LEADTIME	8	12.6 29.6	19.9 22.3	19.4 22.7	
	16	17.4 40.4	27.0 30.9	26.6 31.3	
C (PIX)/C (IN)	2	13.9 32.1	21.3 24.7		
	4	16.1 37.9	25.6 28.5		
LEADTIME	32	13.7 33.2			
	64	16.3 36.8			
C (PIX)/C (IN)	2	6.2 15.0			
	4	7.6 17.0			
LEADTIME	32	7.4 18.1			
	64	8.7 19.8			

Table F4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		109.2	PERCENT OF TOTAL COST=		14.4	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	4	99	32	64	2	8
	16				4	16
LEADTIME	2	56.0	44.0	48.7	51.3	41.9
	4				54.0	58.1
C (FIX)/C (IN)	32	23.0	18.9	20.3	21.6	19.1
	64	33.0	25.1	28.4	29.7	22.8
LEADTIME	2	25.8	20.2	22.4	23.6	26.9
	4	30.2	23.8	26.3	27.7	31.2
C (FIX)/C (IN)	32	26.8	21.9			
	64	29.2	22.1			
LEADTIME	2	12.2	10.2			
	4	13.6	10.0			
C (FIX)/C (IN)	32	14.6	11.7			
	64	15.6	12.1			



Table F5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, OPTIMAL CONTROL(DP)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		211.0	PERCENT OF TOTAL COST=		27.8	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	
	4	99	32	64	2	4
						8
						16
						41.1
						58.9

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

4

32

4

9.8

10.7

14.6

15.9

9.2

10.3

14.0

15.5

19.0

21.0

20.5

30.5

19.4

29.5

16.4

24.8

23.6

35.2

24.4

26.6

23.2

25.8

19.0

21.0

28.6

31.4

9.8

10.7

14.6

15.9

9.2

10.3

14.0

15.5

21.0

20.1

30.0

28.8

51.0

49.0

40.0

60.0

47.6

52.4

19.7

21.5

27.9

30.9

19.0

21.0

23.6

35.2

40.0

Table F6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, OPTIMAL CONTROL (DP)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1005

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN
8	4 99	32 64	2 4	8 16
16	0.1916 0.0095	0.1000 0.1010	0.1010 0.1000	0.0984 0.1026

0.1875 0.0092	0.0971 0.0996	0.0994 0.0974
0.1956 0.0097	0.1028 0.1024	0.1026 0.1027

LEADTIME

0.1926 0.0094	0.1001 0.1019
0.1906 0.0095	0.0999 0.1001

C (FIX)/C (IN)

0.1905 0.0095	0.1927 0.0094
---------------	---------------

LEADTIME C (FIX)/C (IN)

2 32	0.1905 0.0096
2 64	0.1946 0.0092
4 32	0.1904 0.0094
4 64	0.1907 0.0096

Table F7  
 SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 STATIONARY MODEL

FULL INFORMATION, OPTIMAL CONTROL (DP)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0110

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME			MEAN
	4	99	32	64	2	4	8	
8	0.1593	0.0051	0.0108	0.0113	0.0102	0.0119	0.0139	0.0096
16	0.1967	0.0065	0.0135	0.0143	0.0127	0.0151		
LEADTIME	0.1406	0.0043	0.0094	0.0098	0.0089	0.0103		
2	0.1466	0.0046	0.0099	0.0104				
4	0.1720	0.0055	0.0116	0.0122				
C (FIX)/C (IN)	0.1525	0.0050						
32	0.1661	0.0051						
64								

LEADTIME	C (FIX)/C (IN)
2	0.1388 0.0047
2	0.1543 0.0046
4	0.1662 0.0054
4	0.1779 0.0056

Table F8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, OPTIMAL CONTROL (DP)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM = 0.2884

MEAN	C (OUT) / C (IN)			C (FIX) / C (IN)			LEADTIME		MEAN
	4	99		32	64		2	4	
8	0.2745	0.3024		0.3295	0.2473		0.2949	0.2819	0.2369
16	0.2268	0.2471		0.2696	0.2043		0.2422	0.2317	
	0.3221	0.3576		0.3894	0.2904		0.3476	0.3322	
LEADTIME									
2	0.2821	0.3076		0.3386	0.2511				
4	0.2668	0.2971		0.3204	0.2435				
C (FIX) / C (IN)									
32	0.3133	0.3456							
64	0.2356	0.2591							
LEADTIME C (FIX) / C (IN)									
2	0.3238	0.3533							
4	0.2404	0.2619							
8	0.3028	0.3379							
16	0.2308	0.2562							

0.2369 0.3399



## Appendix G

### Multi-item Data for the Power Approximation

#### Stationary Model

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled Approximately Optimally with  
Full Information.

	<u>page</u>
Table G1 Sources of Expected Total Cost	G1
G3 Sources of Aggregate Period-End Inventory	G2
G4 Sources of Aggregate Backlog Cost	G3
G5 Sources of Aggregate Replenishment Cost	G4
G6 Backlog Frequency	G5
G7 Weighted Proportion of Demand Backlogged	G6
G8 Replenishment Frequency	G7
G15 Sources of Total Cost (% Excess Over DP)	G8
G16 Sources of Aggregate Period-End Inventory(% Excess Over DP)	G9
G17 Sources of Aggregate Backlog Cost (% Excess Over DP)	G10
G18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	G11
G19 Backlog Frequency (% Excess Over DP)	G12
G20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	G13
G21 Replenishment Frequency (% Excess Over DP)	G14

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Table G1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

OVERALL AGGREGATE FOR SYSTEM=		760.7		SOURCES OF TOTAL COST		MEAN
LEADTIME	C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN	
2	32	4 99	32 64	2 4	8 16	
4	64	38.6 61.4	45.2 54.8	47.4 52.6	41.8 58.2	
8		16.1 25.8	19.0 22.9	19.8 22.0		
16		22.6 35.6	26.3 31.9	27.6 30.6		
32		18.5 28.9	21.2 26.1			
64		20.1 32.5	24.0 28.7			
128		17.0 28.2				
256		21.6 33.2				
512		8.1 13.2				
1024		10.4 15.7				
2048		9.0 15.0				
4096		11.2 17.5				

Table G3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		440.0		PERCNT OF TOTAL COST=		57.8		MEAN
		C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		
		4	99	32	64	2	4	8 16
		28.2	71.8	47.7	52.3	46.2	53.8	41.5 58.5
MEAN								
	8	11.5	30.0	19.8	21.8	19.1	22.4	
	16	16.7	41.8	27.9	30.5	27.1	31.4	
LEADTIME								
	2	13.3	32.9	21.9	24.2			
	4	14.9	38.9	25.8	28.1			
C (FIX) / C (IN)								
	32	13.2	34.5					
	64	15.1	37.2					
LEADTIME C (FIX) / C (IN)								
	2	6.2	15.7					
	2	7.1	17.1					
	4	7.0	18.8					
	4	8.0	20.1					

Table G4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		109.1	PERCENT OF TOTAL COST=		14.3	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	4	99	32	64	2	4
	8					8
LEADTIME	16					16
	2	58.5	41.5	45.4	54.6	43.7
C (FIX)/C (IN)	32				54.4	56.3
	64					
LEADTIME	2	25.2	18.5	20.1	23.6	20.3
	4	33.2	23.1	25.3	31.0	23.4
C (FIX)/C (IN)	32					25.3
	64					31.0
LEADTIME	2	26.2	19.4	20.6	24.9	20.6
	4	32.3	22.1	24.8	29.6	24.9
C (FIX)/C (IN)	32					
	64					
LEADTIME	2	26.9	18.5			
	4	31.6	23.0			
C (FIX)/C (IN)	32					
	64					
LEADTIME	2	11.8	8.8			
	4	14.3	10.6			
C (FIX)/C (IN)	32					
	64					
LEADTIME	2	15.1	9.7			
	4	17.3	12.4			



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NORTH CAROLINA UNIV AT CHAPEL HILL GRADUATE SCHOOL OF--ETC F/G 15/5  
(S,S) INVENTORY POLICIES IN A NONSTATIONARY DEMAND ENVIRONMENT.--ETC(U)  
APR 77 R L KAUFMAN.

DAAG29-76-G-0323

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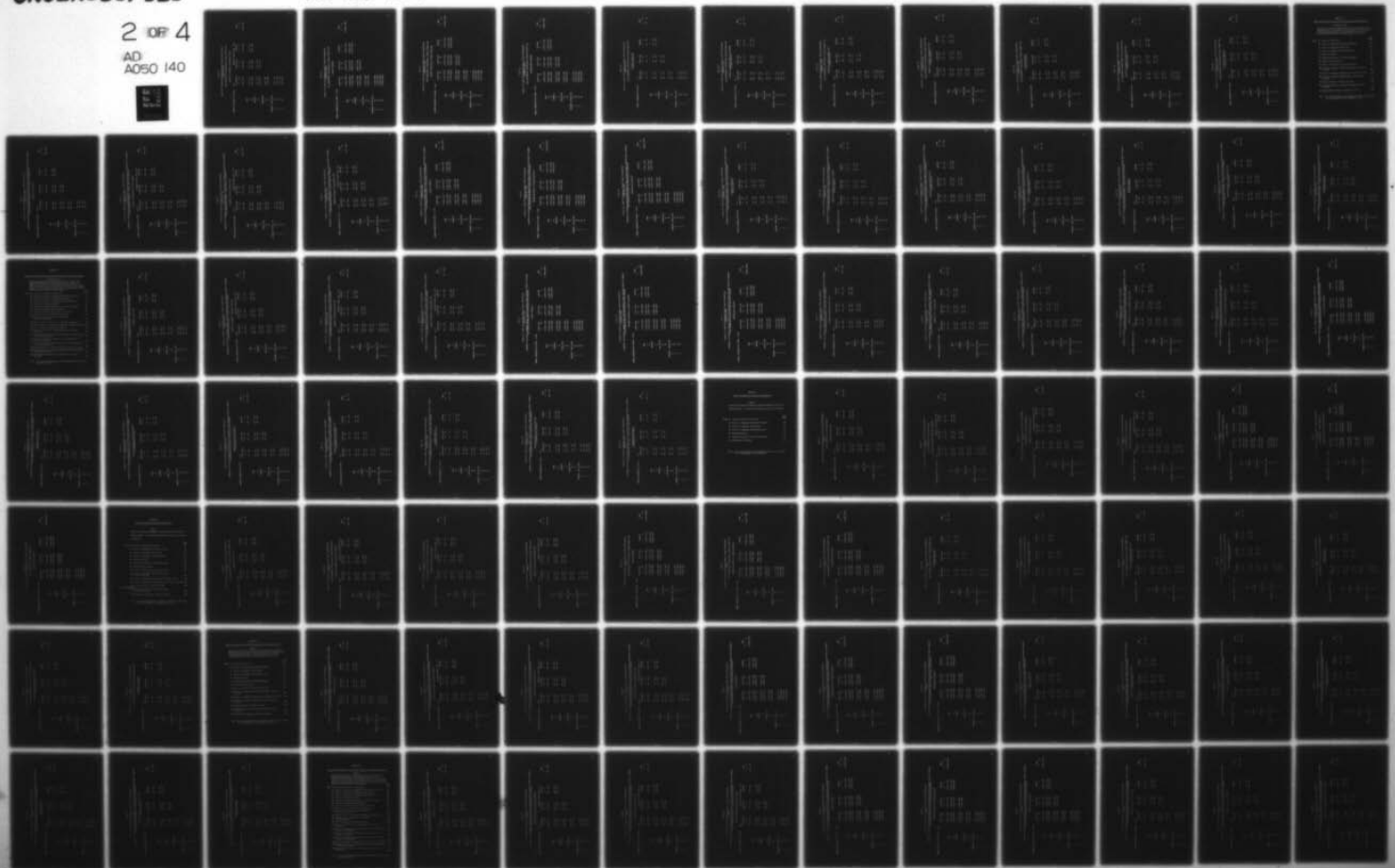
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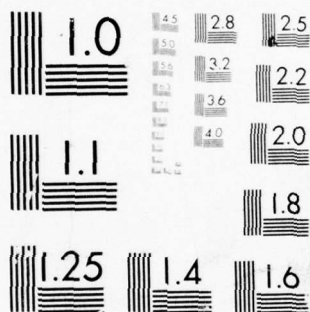
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MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

Table G5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		211.7	PERCENT OF TOTAL COST= 27.8			MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	
		4 99	32 64	2 4		8 16
		50.0 50.0	40.0 60.0	50.8 49.2		41.5 58.5
MEAN						
8		20.7 20.7	16.7 24.8	21.1 20.4		
16		29.3 29.3	23.3 35.2	29.7 28.8		
LEADTIME						
2		25.4 25.4	20.1 30.6			
4		24.6 24.6	19.9 29.4			
C (FIX)/C (IN)						
32		20.0 20.0				
64		30.0 30.0				
LEADTIME C (FIX)/C (IN)						
2		10.1 10.1				
2		15.3 15.3				
4		9.9 9.9				
4		14.7 14.7				

Table G6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1047

MEAN  
8 16  
0.1060 0.1034

LEADTIME  
2 4  
0.1029 0.1065  
0.1038 0.1081  
0.1019 0.1049

C (FIX) / C (IN)  
32 64  
0.1009 0.1085  
0.1028 0.1091  
0.0990 0.1078  
0.0982 0.1075  
0.1036 0.1095

C (OUT) / C (IN)  
4 99  
0.2004 0.0090  
0.2029 0.0091  
0.1980 0.0089  
0.1966 0.0091  
0.2043 0.0088  
0.1937 0.0082  
0.2072 0.0097

MEAN

8

16

LEADTIME

2

4

C (FIX) / C (IN)

32

64

LEADTIME C (FIX) / C (IN)

2

32

2

64

4

32

4

64

0.1880 0.0084  
0.2052 0.0098  
0.1992 0.0080  
0.2093 0.0096



Table G7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0110

MEAN	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN	
				8	16
0.1661	0.0088	0.0100	0.0120	0.0101	0.0120
0.2151	0.0064	0.0133	0.0156	0.0134	0.0155
0.1415	0.0040	0.0084	0.0103	0.0084	0.0103
0.1486	0.0045	0.0091	0.0110		
0.1836	0.0051	0.0109	0.0131		
0.1528	0.0043				
0.1794	0.0053				
0.1345	0.0040				
0.1627	0.0049				
0.1711	0.0045				
0.1961	0.0057				

LEADTIME C(FIX)/C(IN)

2	32
2	64
4	32
4	64

C(FIX)/C(IN)

32	64
----	----

LEADTIME

2	4
---	---

MEAN

8	16
---	----

0.0145 0.0093

Table G8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2894

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN	
				8	16
0.2894	0.2894	0.3308 0.2481	0.2932 0.2856	0.2406	0.3383
0.2406	0.2406	0.2759 0.2052	0.2432 0.2379		
0.3383	0.3383	0.3856 0.2910	0.3433 0.3333		
0.2932	0.2932	0.3331 0.2534			
0.2856	0.2856	0.3284 0.2429			
0.3308	0.3308				
0.2481	0.2481				
0.3331	0.3331				
0.2534	0.2534				
0.3284	0.3284				
0.2429	0.2429				

LEADTIME C (FIX)/C (IN)

2	32
2	64
4	32
4	64



Table G16  
 SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 STATIONARY MODEL  
 FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE PERIOD-END INVENTORY  
 (% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		0.1		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16		
		-5.7	2.6	1.9	-1.4	0.6	-0.2	-1.3	1.2		
MEAN											
	8	-8.1	1.6	-0.4	-2.2	-1.6	-1.1				
	16	-4.0	3.4	3.6	-0.9	2.1	0.4				
LEADTIME											
	2	-4.0	2.5	3.1	-1.7						
	4	-7.1	2.7	0.9	-1.2						
C (FIX)/C (IN)											
	32	-3.5	4.2								
	64	-7.6	1.3								
LEADTIME C (FIX)/C (IN)											
	2	-0.5	4.6								
	2	-7.0	0.7								
	4	-6.0	3.8								
	4	-8.1	1.8								





Table G18  
 SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 STATIONARY MODEL  
 FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

OVERALL AGGREGATE FOR SYSTEM=		0.3		SOURCES OF AGGREGATE REPLENISHMENT COST (% EXCESS OVER DP)		LEADTIME		MEAN
C (OUT)/C (IN)		C (FIX)/C (IN)		C (FIX)/C (IN)		C (FIX)/C (IN)		
4	99	32	64	2	4	8	16	
5.4	-4.3	0.4	0.3	-0.1	0.8	1.2	-0.3	
5.8	-3.0	2.3	0.5	0.6	1.8			
5.2	-5.1	-1.0	0.2	-0.7	0.2			
4.4	-4.3	-1.6	0.9					
6.5	-4.3	2.5	-0.3					
5.6	-4.3							
5.3	-4.2							
2.9	-5.7							
5.4	-3.3							
8.5	-2.8							
5.2	-5.2							

OVERALL AGGREGATE FOR SYSTEM=		0.3		SOURCES OF AGGREGATE REPLENISHMENT COST (% EXCESS OVER DP)		LEADTIME		MEAN
C (OUT)/C (IN)		C (FIX)/C (IN)		C (FIX)/C (IN)		C (FIX)/C (IN)		
4	99	32	64	2	4	8	16	
5.4	-4.3	0.4	0.3	-0.1	0.8	1.2	-0.3	
5.8	-3.0	2.3	0.5	0.6	1.8			
5.2	-5.1	-1.0	0.2	-0.7	0.2			
4.4	-4.3	-1.6	0.9					
6.5	-4.3	2.5	-0.3					
5.6	-4.3							
5.3	-4.2							
2.9	-5.7							
5.4	-3.3							
8.5	-2.8							
5.2	-5.2							

MEAN

8

16

LEADTIME

2

4

C(FIX)/C(IN)

32

64

LEADTIME C(FIX)/C(IN)

2

2

4

4

32

64

32

64

Table G19

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

BACKLOG FREQUENCY  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		4.2															
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		LEADTIME		MEAN							
		4	99	32	64	2	4	2	4	8	16						
		4.6	-5.3	0.9	7.4	1.9	6.5	1.9	6.5	7.7	0.8						
		8.2	-1.9	5.8	9.5	4.5	11.0	4.5	11.0								
		1.2	-8.5	-3.7	5.3	-0.7	2.2	-0.7	2.2								
		2.1	-3.2	-1.8	5.5												
		7.2	-7.4	3.7	9.3												
		1.7	-13.9														
		7.6	3.5														
		-1.3	-12.4														
		5.4	6.5														
		4.6	-15.4														
		9.8	0.5														

LEADTIME C (FIX)/C (IN)

2 32  
2 64  
4 32  
4 64

C (FIX)/C (IN)

32 64

LEADTIME

2 4

MEAN

8 16







## Appendix H

### Multi-item Data for the (24,24) Statistical Power Approximation

#### Stationary Model

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled with Statistical Information from  
a 24-Period Demand History, with Revision Every 24 Periods,  
Using Regression Estimates of Demand Means and Variances.

	<u>page</u>
Table H1 Sources of Total Cost	H1
H3 Sources of Aggregate Period-End Inventory	H2
H4 Sources of Aggregate Backlog Cost	H3
H5 Sources of Aggregate Replenishment Cost	H4
H6 Backlog Frequency	H5
H7 Weighted Proportion of Demand Backlogged	H6
H8 Replenishment Frequency	H7
H15 Sources of Total Cost (% Excess Over DP)	H8
H16 Sources of Aggregate Period-End Inventory (% Excess Over DP)	H9
H17 Sources of Aggregate Backlog Cost (% Excess Over DP)	H10
H18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	H11
H19 Backlog Frequency (% Excess Over DP)	H12
H20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	H13
H21 Replenishment Frequency (% Excess Over DP)	H14

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices K and L.

Table H1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 813.4				SOURCES OF TOTAL COST			
MEAN	C (OUT) / C (IN)	C (FIX) / C (IN)	LEADTIME	MEAN		LEADTIME	MEAN
				4	99	2	4
8	37.4	62.6	45.6	54.4	46.8	53.2	42.0
16	15.6	26.4	19.2	22.7	19.6	22.4	16
LEADTIME	21.8	36.2	26.4	31.6	27.1	30.9	
2	17.7	29.0	21.2	25.6			
4	19.6	33.6	24.5	28.8			
C (FIX) / C (IN)	16.6	29.1					
32	20.8	33.6					
64							
LEADTIME C (FIX) / C (IN)	7.8	13.4					
2	9.9	15.6					
4	8.8	15.7					
64	10.8	17.9					

Table H3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		462.9	PERCENT OF TOTAL COST=		56.9	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
	4	99	32	64	2	4
	28.8	71.2	47.8	52.2	45.7	54.3
						41.7 58.3
MEAN						
8	11.7	30.0	19.9	21.8	19.0	22.7
16	17.1	41.2	27.9	30.4	26.7	31.6
LEADTIME						
2	13.3	32.5	21.7	24.1		
4	15.6	38.7	26.1	28.1		
C (FIX)/C (IN)						
32	13.5	34.3				
64	15.3	36.9				
LEADTIME C (FIX)/C (IN)						
2	6.1	15.5				
2	7.1	16.9				
4	7.3	18.8				
4	8.2	19.9				



Table H4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		137.6	PERCENT OF TOTAL COST=		16.9	MEAN	
LEADTIME	C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	8	16
			32	64			
		46.4	53.6	47.2	52.8	44.0	56.0
MEAN							
8		20.3	23.1	20.7	22.7	19.3	24.1
16		26.1	30.5	26.5	30.1	24.7	31.9
LEADTIME							
2		20.9	23.1	20.7	23.3		
4		25.5	30.5	26.5	29.5		
C (FIX)/C (IN)							
32		21.6	25.6				
64		24.7	28.1				
LEADTIME	C (FIX)/C (IN)						
2		9.6	11.1				
2	64	11.2	12.1				
4	32	12.0	14.5				
4	64	13.5	16.0				

43.4 56.6

Table H5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		212.9	PERCENT OF TOTAL COST=		26.2	MEAN
		C (OUT) / C (IN)	C (FIX) / C (IN)		LEADTIME	
	4	99	32	64	2	4
					8	16
					50.8	49.2
					41.6	58.4

MEAN

8

16

LEADTIME

2

4

C (FIX) / C (IN)

32

64

LEADTIME C (FIX) / C (IN)

2

32

2

64

4

32

4

64

10.2

10.1

15.3

15.2

9.8

9.8

14.8

14.7

20.8

20.7

29.3

29.1

25.5

25.3

24.7

24.5

20.0

19.9

30.1

29.9

16.7

24.8

23.2

35.2

20.3

30.5

19.6

29.6

21.1

20.5

29.7

28.7

Table H6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 0.1043				BACKLOG FREQUENCY			
MEAN	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	MEAN			0.1063 0.1022
				4	32	64	8 16
0.1955 0.0131	0.1955 0.0131	0.1015 0.1070	0.1034 0.1051				
0.1997 0.0130	0.1997 0.0130	0.1036 0.1091	0.1060 0.1067				
0.1913 0.0131	0.1913 0.0131	0.0994 0.1050	0.1009 0.1035				
LEADTIME							
2	0.1944 0.0125	0.1002 0.1066					
4	0.1966 0.0136	0.1028 0.1074					
C (PIX)/C (IN)							
32	0.1904 0.0126						
64	0.2006 0.0135						
LEADTIME C (PIX)/C (IN)							
2	0.1884 0.0121						
2	0.2004 0.0129						
4	0.1924 0.0131						
4	0.2007 0.0141						

Table H7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0139

	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	MEAN	
				2	4
4	99	32	64	8	16
0.1661	0.0078	0.0131	0.0147	0.0122	0.0156
0.2176	0.0100	0.0173	0.0189	0.0161	0.0201
0.1403	0.0066	0.0110	0.0126	0.0103	0.0133
0.1494	0.0067	0.0115	0.0129		
0.1828	0.0088	0.0147	0.0164		
0.1549	0.0074				
0.1773	0.0081				
0.1382	0.0064				
0.1606	0.0070				
0.1716	0.0084				
0.1939	0.0093				



Table H8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 STATIONARY MODEL  
 STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 0.2910				REPLENISHMENT FREQUENCY			
MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN
4	99	32	64	2	4	8	16
0.2918	0.2903	0.3323	0.2497	0.2956	0.2864	0.2825	0.3396
8	0.2431	0.2419	0.2784	0.2065	0.2458	0.2392	
16	0.3405	0.3387	0.3862	0.2929	0.3455	0.3337	
LEADTIME							
2	0.2965	0.2948	0.3379	0.2534			
4	0.2871	0.2858	0.3268	0.2460			
C (FIX)/C (IN)							
32	0.3330	0.3317					
64	0.2505	0.2489					
LEADTIME C (FIX)/C (IN)							
2	0.3389	0.3369					
4	0.2541	0.2527					
8	0.3272	0.3265					
16	0.2470	0.2451					

Table H15

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			SOURCES OF TOTAL COST (% EXCESS OVER DP)			MEAN
7.1						
LEADTIME	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	C (OUT)/C (IN)	C (PIX)/C (IN)	MEAN
2	4	99	2	32	64	8
4	3.6	9.2	5.7	8.1	6.2	7.4
8	3.9	9.6	6.0	8.6	6.4	8.7
16	3.4	9.0	5.4	7.7	6.1	8.1
2	2.6	7.6	6.6	6.6	4.9	
4	4.5	10.7	9.4	9.4	7.4	
32	4.1	10.5				
64	3.2	8.2				
2	3.1	8.8				
2	2.3	6.6				
4	5.0	12.0				
4	4.1	9.6				

Table H16

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

[illegible]

Table H17

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM-		25.9	SOURCES OF AGGREGATE BACKLOG COST (% EXCESS OVER DP)				MEAN	
LEADTIME	C (FIX) / C (IN)	C (OUT) / C (IN)	C (FIX) / C (IN)		LEADTIME		8	16
			32	64	2	4		
2	32	4.3	53.5	21.9	29.7	20.5	30.4	22.7
4	64	10.7	54.5	28.5	32.2	27.2		
8	16	-0.2	52.8	17.2	27.9	15.7		
16	2	1.9	44.1	16.3	24.4	33.1		
32	4	6.2	61.5	26.7	34.2	28.7		
64		1.6	46.8					
		6.7	60.1					
2	32	-0.5	36.4					
4	64	4.1	52.0					
8	16	3.3	55.9					
16	2	9.0	66.9					
32	4							
64								



Table H18

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=				SOURCES OF AGGREGATE REPLISHMENT COST (% EXCESS OVER DP)			
		0.9					
		C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	MEAN		
4	99	32	64	2	4	8	16
6.3	-4.0	0.9	1.0	0.4	1.4	2.0	0.2
MEAN							
8	6.9	-2.5	3.3	1.1	1.2	2.8	
16	5.9	-5.0	-0.8	0.9	-0.1	0.5	
LEADTIME							
2	5.3	-4.0	-0.2	0.9			
4	7.4	-4.0	2.0	1.0			
C (PIX)/C (IN)							
32	6.3	-4.0					
64	6.3	-3.9					
LEADTIME C (PIX)/C (IN)							
2	4.6	-4.7					
2	5.7	-3.5					
4	8.1	-3.4					
4	7.0	-4.3					









## Appendix I

### Multi-item Forecasts for the (24,24) Statistical Power Approximation

#### Stationary Model

Forecasting Properties of Inventory System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information, Revision Taking Place Every 24 Periods Using a 24-Period Demand History and Regression Estimates of Demand Means and Variances. Forecasts Made at Each Revision Using a 24-Period Demand History.

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Table I1 Sources of Forecast of Total Cost	I1
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I5 Forecast of Backlog Frequency	I5
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I19 Weighted Proportion of Demand Backlogged (% Underestimate of Actual by Forecast)	I19
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Note: The corresponding appendices in MacCormick (1974), are his Appendices T and U.

Table II

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 STATIONARY MODEL  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 739.8				SOURCES OF TOTAL COST			
	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	MEAN			
				4	99	2	4
	39.0	61.0		45.1	54.9	47.9	52.1
						41.8	58.2
MEAN							
8	16.3	25.5		18.9	22.9	20.0	21.8
16	22.7	35.5		26.2	32.0	27.9	30.3
LEADTIME							
2	18.8	29.1		21.4	26.6		
4	20.2	31.9		23.7	28.4		
C (PIX)/C (IN)							
32	17.1	27.9					
64	21.9	33.0					
LEADTIME C (PIX)/C (IN)							
2	8.2	13.2					
2	10.6	15.9					
4	8.9	14.8					
4	11.2	17.1					

Table I2

SYSTEM OP 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 449.5				SOURCES OF AGGREGATE PERIOD-END INVENTORY			
		C (OUT)/C (IN)	C (FIX)/C (IN)	PERCENT OF TOTAL COST= 60.8		MEAN	
		4	99	32	64	2	4
		28.0	72.0	47.7	52.3	46.1	53.9
							41.7 58.3
MEAN							
8		11.4	30.3	19.9	21.8	19.2	22.5
16		16.6	41.7	27.8	30.5	26.9	31.4
LEADTIME							
2		13.1	33.0	21.8	24.3		
4		14.9	39.0	23.9	28.0		
C (FIX)/C (IN)							
32		13.0	34.7				
64		15.0	37.3				
LEADTIME C (FIX)/C (IN)							
2		6.0	15.8				
4		7.1	17.2				
8		7.0	18.9				
16		7.9	20.1				

Table I3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL  
FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST					
OVERALL AGGREGATE FOR SYSTEM=		78.2	PERCENT OF TOTAL COST= 10.6		MEAN
	C (OUT)/C (IN)	4 99	C (FIX)/C (IN)		LEADTIME
			32 64	2 4	
					8 16
		72.5 27.5	43.7 56.3	50.7 49.3	43.6 56.4
MEAN					
8		32.2 11.3	18.7 24.9	22.2 21.4	
16		40.2 16.2	25.0 31.4	28.5 27.9	
LEADTIME					
2		34.1 16.6	21.8 28.8		
4		38.4 10.9	21.8 27.5		
C (FIX)/C (IN)					
32		33.0 10.7			
64		39.5 16.9			
LEADTIME C (FIX)/C (IN)					
2		15.5 6.4			
2		18.6 10.2			
4		17.6 4.3			
4		20.9 6.7			



Table I4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST				
OVERALL AGGREGATE FOR SYSTEM=		212.1	PERCENT OF TOTAL COST= 28.7	
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME
		4 99	32 64	2 4 8 16
MEAN				
8		50.1 49.9	40.0 60.0	50.8 49.2
16				41.4 58.6
LEADTIME				
2		20.8 20.6	16.7 24.7	21.0 20.4
4		29.3 29.3	23.3 35.3	29.8 28.8
		25.4 25.4	20.3 30.5	
		24.7 24.6	19.7 29.5	
C (FIX)/C (IN)				
32		20.0 20.0		
64		30.0 30.0		
LEADTIME C (FIX)/C (IN)				
2		10.2 10.2		
2		15.2 15.2		
4		9.9 9.8		
4		14.8 14.7		

Table I5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 STATIONARY MODEL  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 0.0983				BACKLOG FREQUENCY			
C (OUT)/C (IN)		C (PIX)/C (IN)		LEADTIME		MEAN	
4	99	32	64	2	4	8	16
0.1912 0.0055		0.0949 0.1018		0.0989 0.0978		0.1002 0.0965	
0.1953 0.0052		0.0965 0.1039		0.0995 0.1009			
0.1871 0.0058		0.0933 0.0996		0.0982 0.0947			
LEADTIME							
2		0.1910 0.0067		0.0961 0.1016			
4		0.1914 0.0043		0.0936 0.1020			
C (PIX)/C (IN)							
32		0.1852 0.0046					
64		0.1972 0.0064					
LEADTIME C (PIX)/C (IN)							
2		0.1861 0.0061					
2		0.1959 0.0072					
4		0.1843 0.0030					
4		0.1984 0.0055					

Table I6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0079

MEAN	C (OUT) / C (IN)		C (PIX) / C (IN)		LEADTIME		MEAN	
	4	99	32	64	2	4	8	16

0.0103 0.0067

MEAN

8

16

LEADTIME

2

4

C (PIX) / C (IN)

32

64

LEADTIME C (PIX) / C (IN)

2

32

2

64

4

32

4

64

0.1260 0.0021

0.1515 0.0034

0.1430 0.0014

0.1699 0.0022

0.0080 0.0078

0.0105 0.0102

0.0068 0.0066

0.0069 0.0089

0.0089 0.0118

0.0059 0.0075

0.0069 0.0091

0.0069 0.0087

0.1345 0.0018

0.1607 0.0028

0.1476 0.0023

0.1969 0.0028

0.1229 0.0020

0.1387 0.0027

0.1564 0.0018

Table I7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 STATIONARY MODEL  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2899

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
0.2904	0.2895	0.3314	0.2485	0.2945	0.2854	0.2405	0.3394
0.2412	0.2398	0.2763	0.2047	0.2442	0.2368		
0.3396	0.3392	0.3864	0.2924	0.3448	0.3340		
0.2947	0.2943	0.3367	0.2523				
0.2861	0.2847	0.3260	0.2448				
0.3320	0.3307						
0.2488	0.2483						
0.3371	0.3363						
0.2523	0.2523						
0.3269	0.3251						
0.2453	0.2443						

LEADTIME C (FIX)/C (IN)

2 32  
 2 64  
 4 32  
 4 64







Table II0

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF S.D. OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=										20.1	
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN			
		4	99	32	64	2	4	8	16		
		6.8	18.9	14.5	13.9	11.9	16.2	12.5	15.7		
		4.2	11.8	9.0	8.7	7.5	10.1				
		5.4	14.7	11.4	10.8	9.3	12.6				
		3.6	11.3	8.6	8.2						
		5.8	15.1	11.7	11.2						
		4.8	13.6								
		4.8	13.0								
		2.6	8.2								
		2.5	7.9								
		4.1	10.9								
		4.1	10.4								

LEADTIME C (FIX) / C (IN)







Table 113

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## S.D. OF BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM = 0.1674

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
8	0.1625	0.0402	0.1193	0.1174	0.1141	0.1224	0.1245
16	0.1213	0.0283	0.0870	0.0891	0.0858	0.0903	0.1118
LEADTIME	0.1081	0.0286	0.0817	0.0763	0.0753	0.0827	
2	0.1099	0.0308	0.0802	0.0812			
4	0.1197	0.0258	0.0883	0.0848			
C (FIX) / C (IN)							
32	0.1164	0.0262					
64	0.1133	0.0305					
LEADTIME C (FIX) / C (IN)							
2	0.0775	0.0209					
2	0.0779	0.0226					
4	0.0869	0.0158					
4	0.0823	0.0204					

Table I14

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF TOTAL COST  
(% UNDERESTIMATE OF ACTUAL)

OVERALL AGGREGATE FOR SYSTEM=										9.0		
					C (OUT)/C (IN)	C (PIX)/C (IN)		LEADTIME	MEAN			
					4	99	32	64	2	4	8	16
					5.0	11.5	10.2	8.1	6.8	11.1	9.4	8.8
MEAN												
8					4.8	12.1	10.8	8.3	7.1	11.4		
16					5.1	11.0	9.8	7.9	6.5	10.8		
LEADTIME												
2					3.4	8.8	8.1	5.6				
4					6.5	13.7	12.0	10.2				
C (PIX)/C (IN)												
32					5.9	12.7						
64					4.3	10.4						
LEADTIME												
2					4.2	10.4						
2					2.7	7.5						
4					7.5	14.6						
4					5.7	13.0						

MEAN

8

16

LEADTIME

2

4

C (PIX)/C (IN)

32

64

LEADTIME C (PIX)/C (IN)

2

32

2

64

4

32

4

64









Table I18

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=		5.7	BACKLOG FREQUENCY (% UNDERESTIMATE OF ACTUAL)			LEADTIME		MEAN
		C (OUT)/C (IN)	C (PIX)/C (IN)					
MEAN	4	99	32	64		2	4	8 16
	2.2	58.2	6.5	4.9		4.4	6.9	5.8 5.6
	2.2	60.4	6.9	4.7		6.1	5.5	
LEADTIME	2.2	55.9	6.1	5.1		2.7	8.4	
	1.7	46.6	4.1	4.8				
	2.6	68.7	8.9	5.1				
C (PIX)/C (IN)		2.7	63.7					
		1.7	53.0					
LEADTIME C (PIX)/C (IN)	2	32	1.2	49.3				
	2	64	2.2	44.2				
	4	32	4.2	77.0				
	4	64	1.2	61.0				





Table 120

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
STATIONARY MODEL

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			REPLENISHMENT FREQUENCY (% UNDERESTIMATE OF ACTUAL)			MEAN	
0.4			C (OUT) / C (IN)			LEADTIME	
MEAN	8	16	4	99	C (FIX) / C (IN)	2	4
			0.5	0.3	0.3	0.4	0.4
LEADTIME	2	4	0.8	0.9	0.8	0.9	0.7
			0.3	-0.2	-0.0	0.2	1.0
C (FIX) / C (IN)	32	64	0.6	0.2	0.3	0.4	0.2
			0.3	0.4	0.3	0.5	-0.1
LEADTIME	2	4	0.3	0.3	0.3	0.3	0.8
			0.5	0.3	0.3	0.5	0.1
C (FIX) / C (IN)	32	64	0.3	0.3	0.3	0.3	0.8
			0.7	0.3	0.3	0.3	0.1
LEADTIME	2	4	0.5	0.2	0.3	0.3	0.8
			0.7	0.2	0.3	0.3	0.1
C (FIX) / C (IN)	32	64	0.3	0.3	0.3	0.3	0.8
			0.7	0.3	0.3	0.3	0.1
LEADTIME	2	4	0.5	0.2	0.3	0.3	0.8
			0.7	0.2	0.3	0.3	0.1
C (FIX) / C (IN)	32	64	0.3	0.3	0.3	0.3	0.8
			0.7	0.3	0.3	0.3	0.1

## Appendix J

### Multi-item Data for the Power Approximation

#### Model I

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled Optimally with Full Information.

	<u>page</u>
Table J1 Sources of Expected Total Cost	J1
J3 Sources of Aggregate Period-End Inventory	J2
J4 Sources of Aggregate Backlog Cost	J3
J5 Sources of Aggregate Replenishment Cost	J4
J6 Backlog Frequency	J5
J7 Weighted Proportion of Demand Backlogged	J6
J8 Replenishment Frequency	J7

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.







Table J4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		105.7	PERCENT OF TOTAL COST=		14.2	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	
MEAN	8	4 99	32 64	2 4	8 16	
	16	56.8 43.2	48.7 51.3	45.5 54.5	42.9 57.1	
LEADTIME	2	24.1 18.8	21.0 21.9	19.6 23.3		
	4	32.7 24.4	27.7 29.4	25.9 31.2		
C (FIX)/C (IN)	32	25.8 19.7	22.2 23.3			
	64	31.0 23.5	26.5 28.0			
LEADTIME C (FIX)/C (IN)	2	27.3 21.4				
	4	29.5 21.8				
2	32	12.4 9.8				
	64	13.4 9.9				
4	32	14.8 11.7				
	64	16.1 11.8				

Table J5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		208.0	PERCENT OF TOTAL COST=		27.9	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	8	4	32	64	2	4
	16	47.3	52.7	39.8	60.2	51.1
LEADTIME	2	19.2	21.2	16.5	23.9	20.6
	4	28.1	31.6	23.4	36.3	30.5
C (FIX)/C (IN)	32	24.2	26.9	20.3	30.8	
	64	23.1	25.8	19.5	29.4	
LEADTIME	2	18.8	21.1			
	4	28.5	31.7			
C (FIX)/C (IN)	32	9.6	10.8			
	64	14.6	16.1			
LEADTIME	2	9.2	10.3			
	4	13.9	15.5			
						40.4
						59.6

Table J6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, OPTIMAL CONTROL (DP)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOF SYSTPM= 0.1005

MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	LEADTIME		MEAN
			2	4	
8	0.1919 0.0092	0.1003 0.1008	0.1003 0.1008		0.1000 0.1011
16	0.1908 0.0091	0.0998 0.1001	0.0997 0.1003		
	0.1929 0.0094	0.1008 0.1015	0.1009 0.1014		
	0.1914 0.0092	0.1003 0.1003			
	0.1924 0.0093	0.1003 0.1013			

C (FIX) /C (IN)

32	0.1914 0.0092
64	0.1924 0.0092

LEADTIME C (FIX) /C (IN)

2	0.1914 0.0092
4	0.1914 0.0092
8	0.1913 0.0093
16	0.1914 0.0093

Table J7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

PULL INFORMATION, OPTIMAL CONTROL (DP)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM = 0.0107

MEAN	C (OUT) / C (IN)	C (FIX) / C (IN)	LEADTIME			MEAN
			2	4	8	
8	0.1564	0.0048	0.0104	0.0110	0.0097	0.0116
16	0.1988	0.0063	0.0135	0.0140	0.0126	0.0150
LEADTIME	0.1352	0.0041	0.0089	0.0094	0.0083	0.0100
2	0.1422	0.0044	0.0095	0.0100		
4	0.1706	0.0052	0.0113	0.0120		
C (FIX) / C (IN)	0.1502	0.0048				
32	0.1626	0.0048				
64						

LEADTIME	C (FIX) / C (IN)
2	0.1368
4	0.1676
8	0.1635
16	0.1777

0.0138 0.0092



Table J8  
SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, OPTIMAL CONTROL (DP)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOF SYSTEM= 0.2841

MEAN	C (OUT) / C (IN)	C (FIX) / C (IN)			LEADTIME		MEAN
		4	32	64	2	4	
8	0.2683	0.2999	0.3237	0.2444	0.2901	0.2781	0.2309
16	0.2186	0.2431	0.2676	0.1941	0.2356	0.2262	0.3373
	0.3179	0.3566	0.3799	0.2947	0.3446	0.3299	

LEADTIME

2	0.2740	0.3061	0.3302	0.2499
4	0.2625	0.2936	0.3172	0.2389

C (FIX) / C (IN)

32	0.3051	0.3424
64	0.2315	0.2573

LEADTIME C (FIX) / C (IN)

2	0.3104	0.3501
4	0.2376	0.2621
32	0.2997	0.3347
64	0.2253	0.2525

## Appendix K

### Multi-item Data for the Power Approximation

#### Model I

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Approximately Optimally with Full  
Information.

	<u>page</u>
Table K1 Sources of Expected Total Cost	K1
K3 Sources of Aggregate Period-End Inventory	K2
K4 Sources of Aggregate Backlog Cost	K3
K5 Sources of Aggregate Replenishment Cost	K4
K6 Backlog Frequency	K5
K7 Weighted Proportion of Demand Backlogged	K6
K8 Replenishment Frequency	K7
K15 Sources of Total Cost (% Excess Over DP)	K8
K16 Sources of Aggregate Period-End Inventory (% Excess Over DP)	K9
K17 Sources of Aggregate Backlog Cost (% Excess Over DP)	K10
K18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	K11
K19 Backlog Frequency (% Excess Over DP)	K12
K20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	K13
K21 Replenishment Frequency (% Excess Over DP)	K14

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Table KI

SYSTEM OP 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

OVERALL AGGREGATE FOR SYSTEM=		757.0	SOURCES OF TOTAL COST	
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME
	4	99	32	64
	38.6	61.4	45.2	54.8
MEAN				47.4
	8			52.6
	16			41.9
				58.1
LEADTIME				
	2	16.1	25.9	19.0
	4	22.5	35.5	22.9
				26.2
				31.8
	2	18.5	28.9	21.2
	4	20.1	32.5	26.1
				24.0
				28.7
C (PIX)/C (IN)				
	32	17.0	28.2	
	64	21.6	33.2	
LEADTIME C (PIX)/C (IN)				
	2	8.1	13.2	
	4	10.4	15.7	
	8	8.9	15.0	
	16	11.2	17.5	

Table K3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		445.4	PERCENT OF TOTAL COST= 58.8		MEAN	
LEADTIME	C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	8	16
		4	32	2	4	
		28.5	47.4	46.4	53.6	41.6 58.4
		11.5	30.1	19.8	21.8	
		17.0	41.4	27.7	30.7	
		13.4	33.0	21.8	24.6	
		15.1	38.5	25.7	28.0	
		13.2	34.2			
		15.3	37.3			
		6.1	15.6			
		7.3	17.3			
		7.1	18.6			
		8.0	19.9			



Table K4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		104.7	PERCENT OF TOTAL COST= 13.8		MEAN
		C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	
MEAN	8	4 99	32 64	2 4	8 16
	16	58.3 41.7	46.7 53.3	45.4 54.6	43.8 56.2
LEADTIME	2	25.8 18.0	20.2 23.6	20.4 23.4	
	4	32.5 23.7	26.5 29.6	25.0 31.2	
C(FIX)/C(IN)	32	26.3 19.1	21.4 24.0		
	64	32.0 22.6	25.3 29.3		
LEADTIME C(FIX)/C(IN)	2	27.4 19.4			
	4	30.9 22.3			
LEADTIME C(FIX)/C(IN)	2	12.3 9.1			
	4	14.0 9.9			
LEADTIME C(FIX)/C(IN)	2	15.1 10.2			
	4	16.9 12.4			

Table K5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		206.9	PERCENT OF TOTAL COST= 27.3		MEAN	
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	8	4 99	32 64	2 4	8	16
	16	50.3 49.7	39.6 60.4	50.5 49.5	41.7	58.3
LEADTIME	2	21.0 20.7	16.7 25.0	21.0 20.6		
	4	29.4 29.0	22.9 35.4	29.5 28.9		
C (PIX)/C (IN)	32	25.5 25.0	20.0 30.5			
	64	28.9 24.6	19.6 29.9			
LEADTIME	2	20.0 19.7				
	4	30.4 30.0				
C (PIX)/C (IN)	32	10.1 9.9				
	64	15.3 15.1				
LEADTIME	2	9.9 9.7				
	4	15.0 14.9				

Table K6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

PULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1018

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
	0.1949	0.0087	0.0996	0.1040	0.1010	0.1025	0.1047 0.0988
8	0.2009	0.0085	0.1027	0.1068	0.1055	0.1040	
16	0.1889	0.0088	0.0965	0.1012	0.0965	0.1011	
LEADTIME							
2	0.1934	0.0086	0.0988	0.1033			
u	0.1954	0.0087	0.1004	0.1047			
C (FIX)/C (IN)							
32	0.1910	0.0082					
64	0.1988	0.0092					

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64

0.1892 0.0083

0.1976 0.0089

0.1928 0.0080

0.2000 0.0094

MEAN

8

16

LEADTIME

2

4

0.1047 0.0988

Table K7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0106

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		MEAN
			2	4	
4	99	32	64		8
0.1590	0.0046	0.0099	0.0113	0.0096	0.0139
8	0.2112	0.0060	0.0128	0.0150	0.0149
16	0.1329	0.0039	0.0084	0.0094	0.0079
LEADTIME					0.0089
2	0.1436	0.0042	0.0091	0.0102	
4	0.1744	0.0050	0.0107	0.0124	
C (FIX)/C (IN)					
32	0.1492	0.0043			
64	0.1687	0.0049			
LEADTIME	C (FIX)/C (IN)				
2	0.1341	0.0040			
4	0.1531	0.0044			
8	0.1644	0.0045			
16	0.1844	0.0055			



Table K8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL 1

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM = 0.2821

MFAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN	
	4	99	32	64	2	4	8	16
	0.2842	0.2801	0.3203	0.2439	0.2850	0.2792	0.2359	0.3283
8	0.2375	0.2342	0.2702	0.2016	0.2384	0.2334		
16	0.3308	0.3259	0.3705	0.2862	0.3316	0.3251		

LEADTIME

2	0.2875	0.2825	0.3238	0.2462
4	0.2809	0.2776	0.3169	0.2416

C (FIX)/C (IN)

32	0.3229	0.3177
64	0.2455	0.2424

LEADTIME C (FIX)/C (IN)

2	0.3269	0.3206
2	0.2480	0.2444
4	0.3189	0.3148
4	0.2429	0.2403

Table K15

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF TOTAL COST  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		1.4							
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN	
MEAN	4	99		32	64	2	4	8	16
	1.2	1.5		1.2	1.5	1.6	1.2	1.5	1.3
	1.4	1.6		1.2	1.8	1.7	1.4		
LEADTIME	1.0	1.4		1.3	1.3	1.5	1.1		
	1.2	1.8		1.4	1.7				
	1.2	1.3		1.1	1.4				
C (FIX)/C (IN)		1.0 1.4							
		1.4 1.6							
LEADTIME	2	32							
	4								
	1.1	1.7							
C (FIX)/C (IN)	1.4	1.9							
	0.9	1.2							
	1.4	1.4							











Table K20

SYSTEM OF 16 TERMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=				
	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN
-1.0				
	4 99	32 64	2 4	8 16
	1.6 -4.4	-4.9 2.8	-1.2 -0.7	1.2 -2.6
MPAN				
8	6.3 -5.2	-4.8 7.0	3.3 -0.5	
16	-1.7 -3.7	-5.1 -0.2	-4.6 -0.9	
LEADTIME				
2	1.0 -4.1	-4.4 1.8		
4	2.2 -4.6	-5.4 3.7		
C (FIX)/C (IN)				
32	-0.6 -10.4			
64	3.8 1.6			
LEADTIME C (FIX)/C (IN)				
2	-2.0 -7.4			
2	3.8 -0.9			
4	0.5 -13.0			
4	3.8 3.7			

Table K21

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

REPLENISHMENT FREQUENCY  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		-0.7																	
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		LEADTIME		LEADTIME		LEADTIME		LEADTIME		LEADTIME		LEADTIME	
		4 99		32 64		2 4		2 4		2 4		2 4		2 4		2 4		2 4	
		5.9 -6.6		-1.1 -0.2		-1.7 0.4		-1.7 0.4		-1.7 0.4		-1.7 0.4		-1.7 0.4		-1.7 0.4		-1.7 0.4	
		8.6 -3.6		0.9 3.9		1.2 3.2		1.2 3.2		1.2 3.2		1.2 3.2		1.2 3.2		1.2 3.2		1.2 3.2	
		4.1 -8.6		-2.5 -2.9		-3.8 -1.5		-3.8 -1.5		-3.8 -1.5		-3.8 -1.5		-3.8 -1.5		-3.8 -1.5		-3.8 -1.5	
		4.9 -7.7		-2.0 -1.5															
		7.0 -5.5		-0.1 1.1															
		5.8 -7.2																	
		6.0 -5.8																	
		5.3 -8.4																	
		4.4 -6.7																	
		6.4 -5.9																	
		7.8 -4.8																	

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64

MEAN

8 16

2.2 -2.6



## Appendix L

### Multi-item Data for the (24,24) Statistical Power Approximation

#### Model I

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled with Statistical Information from  
a 24-Period Demand History, with Revision Every 24 Periods,  
Using Regression Estimates of Demand Means and Variances.

	<u>page</u>
Table L1 Sources of Total Cost	L1
L3 Sources of Aggregate Period-End Inventory	L2
L4 Sources of Aggregate Backlog Cost	L3
L5 Sources of Aggregate Replenishment Cost	L4
L6 Backlog Frequency	L5
L7 Weighted Proportion of Demand Backlogged	L6
L8 Replenishment Frequency	L7
L15 Sources of Total Cost (% Excess Over DP)	L8
L16 Sources of Aggregate Period-End Inventory (% Excess Over DP)	L9
L17 Sources of Aggregate Backlog Cost (% Excess Over DP)	L10
L18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	L11
L19 Backlog Frequency (% Excess Over DP)	L12
L20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	L13
L21 Replenishment Frequency (% Excess Over DP)	L14

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices K and L.

Table L1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM= 810.5

	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN
	4 99	32 64	2 4	8 16
	37.3 62.7	45.6 54.4	46.8 53.2	42.1 57.9

## MEAN

8

16

## LEADTIME

2

4

## C (FIX)/C (IN)

32

64

## LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64

7.8 13.3

10.0 15.8

8.8 15.7

10.8 17.9

16.6 29.0

20.8 33.6

17.7 29.1

19.6 33.6

15.6 26.5

21.7 36.1

21.1 25.7

24.5 28.7

19.2 22.9

26.4 31.5

19.7 22.4

27.1 30.8

Table L3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		472.7	PERCENT OF TOTAL COST= 58.3		MEAN	
		C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	
4		99	32	64	2	4
					8	16
28.8		71.2	47.5	52.5	45.9	54.1
					42.0	58.0
MEAN						
8						
11.8		30.1	20.1	21.9	19.3	22.6
16						
17.0		41.0	27.4	30.6	26.5	31.5
LEADTIME						
2						
13.3		32.6	21.5	24.4		
n						
15.6		38.6	26.0	28.1		
C (FIX)/C (IN)						
32						
13.4		34.1				
64						
15.4		37.1				
LEADTIME C (FIX)/C (IN)						
2						
6.0		15.5				
2						
7.2		17.1				
4						
7.4		18.6				
4						
8.2		19.9				

Table L4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL 1

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE POP SYSTEM=		130.3	PERCENT OF TOTAL COST=		16.1	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	8	4 99	32 64	2 4		8 16
	16					
LEADTIME	2	47.3 52.7	48.0 52.0	44.1 55.9		43.6 56.4
	4					
C (FIX)/C (IN)	32	20.6 23.0	20.5 23.2	19.1 24.5		
	64	26.6 29.7	27.6 28.8	25.0 31.4		
LEADTIME C (FIX)/C (IN)	2	21.4 22.7	21.0 23.1			
	4	25.8 30.0	27.0 28.9			
LEADTIME C (FIX)/C (IN)	32	22.4 25.6				
	64	24.9 27.1				
LEADTIME C (FIX)/C (IN)	2	10.1 10.9				
	4	11.3 11.8				
LEADTIME C (FIX)/C (IN)	32	12.3 14.7				
	64	13.6 15.4				



Table L5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL I  
 STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST									
OVERALL AGGREGATE FOR SYSTEM=			207.6	PERCENT OF TOTAL COST=			25.6	MEAN	
LEADTIME	C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)	C (OUT)/C (IN)	LEADTIME	8	16
2	32	64	32	64	2	4	2	4	8
4	32	64	32	64	2	4	2	4	16
8	32	64	32	64	2	4	2	4	16
16	32	64	32	64	2	4	2	4	16
32	32	64	32	64	2	4	2	4	16
64	32	64	32	64	2	4	2	4	16
128	32	64	32	64	2	4	2	4	16
256	32	64	32	64	2	4	2	4	16
512	32	64	32	64	2	4	2	4	16
1024	32	64	32	64	2	4	2	4	16
2048	32	64	32	64	2	4	2	4	16
4096	32	64	32	64	2	4	2	4	16
8192	32	64	32	64	2	4	2	4	16
16384	32	64	32	64	2	4	2	4	16
32768	32	64	32	64	2	4	2	4	16
65536	32	64	32	64	2	4	2	4	16
131072	32	64	32	64	2	4	2	4	16
262144	32	64	32	64	2	4	2	4	16
524288	32	64	32	64	2	4	2	4	16
1048576	32	64	32	64	2	4	2	4	16
2097152	32	64	32	64	2	4	2	4	16
4194304	32	64	32	64	2	4	2	4	16
8388608	32	64	32	64	2	4	2	4	16
16777216	32	64	32	64	2	4	2	4	16
33554432	32	64	32	64	2	4	2	4	16
67108864	32	64	32	64	2	4	2	4	16
134217728	32	64	32	64	2	4	2	4	16
268435456	32	64	32	64	2	4	2	4	16
536870912	32	64	32	64	2	4	2	4	16
1073741824	32	64	32	64	2	4	2	4	16
2147483648	32	64	32	64	2	4	2	4	16
4294967296	32	64	32	64	2	4	2	4	16
8589934592	32	64	32	64	2	4	2	4	16
17179869184	32	64	32	64	2	4	2	4	16
34359738368	32	64	32	64	2	4	2	4	16
68719476736	32	64	32	64	2	4	2	4	16
137438953472	32	64	32	64	2	4	2	4	16
274877906944	32	64	32	64	2	4	2	4	16
549755813888	32	64	32	64	2	4	2	4	16
1099511627776	32	64	32	64	2	4	2	4	16
2199023255552	32	64	32	64	2	4	2	4	16
4398046511104	32	64	32	64	2	4	2	4	16
8796093022208	32	64	32	64	2	4	2	4	16
17592186044416	32	64	32	64	2	4	2	4	16
35184372088832	32	64	32	64	2	4	2	4	16
70368744177664	32	64	32	64	2	4	2	4	16
140737488355328	32	64	32	64	2	4	2	4	16
281474976710656	32	64	32	64	2	4	2	4	16
562949953421312	32	64	32	64	2	4	2	4	16
1125899906842624	32	64	32	64	2	4	2	4	16
2251799813685248	32	64	32	64	2	4	2	4	16
4503599627370496	32	64	32	64	2	4	2	4	16
9007199254740992	32	64	32	64	2	4	2	4	16
18014398509481984	32	64	32	64	2	4	2	4	16
36028797018963968	32	64	32	64	2	4	2	4	16
72057594037927936	32	64	32	64	2	4	2	4	16
144115188075855872	32	64	32	64	2	4	2	4	16
288230376151711744	32	64	32	64	2	4	2	4	16
576460752303423488	32	64	32	64	2	4	2	4	16
1152921504606846976	32	64	32	64	2	4	2	4	16
2305843009213693952	32	64	32	64	2	4	2	4	16
4611686018427387904	32	64	32	64	2	4	2	4	16
9223372036854775808	32	64	32	64	2	4	2	4	16
18446744073709551616	32	64	32	64	2	4	2	4	16
36893488147419103232	32	64	32	64	2	4	2	4	16
73786976294838206464	32	64	32	64	2	4	2	4	16
147573952589676412928	32	64	32	64	2	4	2	4	16
295147905179352825856	32	64	32	64	2	4	2	4	16
590295810358705651712	32	64	32	64	2	4	2	4	16
1180591620717411303424	32	64	32	64	2	4	2	4	16
2361183241434822606848	32	64	32	64	2	4	2	4	16
4722366482869645213696	32	64	32	64	2	4	2	4	16
9444732965739290427392	32	64	32	64	2	4	2	4	16
18889465931478580854784	32	64	32	64	2	4	2	4	16
37778931862957161709568	32	64	32	64	2	4	2	4	16
75557863725914323419136	32	64	32	64	2	4	2	4	16
151115727451828646838272	32	64	32	64	2	4	2	4	16
302231454903657293676544	32	64	32	64	2	4	2	4	16
604462909807314587353088	32	64	32	64	2	4	2	4	16
1208925819614629174706176	32	64	32	64	2	4	2	4	16
2417851639229258349412352	32	64	32	64	2	4	2	4	16
4835703278458516698824704	32	64	32	64	2	4	2	4	16
9671406556917033397649408	32	64	32	64	2	4	2	4	16
19342813113834066795298816	32	64	32	64	2	4	2	4	16
38685626227668133590597632	32	64	32	64	2	4	2	4	16
77371252455336267181195264	32	64	32	64	2	4	2	4	16
154742504910672534362390528	32	64	32	64	2	4	2	4	16
309485009821345068724781056	32	64	32	64	2	4	2	4	16
618970019642690137449562112	32	64	32	64	2	4	2	4	16
1237940039285380274899124224	32	64	32	64	2	4	2	4	16
2475880078570760549798248448	32	64	32	64	2	4	2	4	16
4951760157141521099596496896	32	64	32	64	2	4	2	4	16
9903520314283042199192993792	32	64	32	64	2	4	2	4	16
19807040628566084398385987584	32	64	32	64	2	4	2	4	16
39614081257132168796771975168	32	64	32	64	2	4	2	4	16
79228162514264337593543950336	32	64	32	64	2	4	2	4	16
158456325028528675187087900672	32	64	32	64	2	4	2	4	16
316912650057057350374175801344	32	64	32	64	2	4	2	4	16
633825300114114700748351602688	32	64	32	64	2	4	2	4	16
1267650600228229401496703205376	32	64	32	64	2	4	2	4	16
2535301200456458802993406410752	32	64	32	64	2	4	2	4	16
5070602400912917605986812821504	32	64	32	64	2	4	2	4	16
10141204801825835211973625643008	32	64	32	64	2	4	2	4	16
20282409603651670423947251286016	32	64	32	64	2	4	2	4	16
40564819207303340847894502572032	32	64	32	64	2	4	2	4	16
81129638414606681695789005144064	32	64	32	64	2	4	2	4	16
162259276829213363391578010288128	32	64	32	64	2	4	2	4	16
324518553658426726783156020576256	32	64	32	64	2	4	2	4	16
649037107316853453566312041152512	32	64	32	64	2	4	2	4	16
1298074214633706907132624082305024	32	64	32	64	2	4	2	4	16
2596148429267413814265248164610048	32	64	32	64	2	4	2	4	16
5192296858534827628530496329220096	32	64	32	64	2	4	2	4	16
10384593717069655257060992658440192	32	64	32	64	2	4	2	4	16
20769187434139310514121985316880384	32	64	32	64	2	4	2	4	16
41538374868278621028243970633760768	32	64	32	64	2	4	2	4	16
83076749736557242056487941267521536	32	64	32	64	2	4	2	4	16
166153499473114484112975882535043072	32	64	32	64	2	4	2	4	16
332306998946228968225951765070086144	32	64	32	64	2	4	2	4	16
664613997892457936451903530140172288	32	64	32	64	2	4	2	4	16
1329227995784915872903807060280344576	32	64	32	64	2	4	2	4	16
2658455991569831745807614120560689152	32	64	32	64	2	4	2	4	16
5316911983139663491615228241121378304	32	64	32	64	2	4	2	4	16
10633823966279326983230456482242756608	32	64	32	64	2	4	2	4	16
21267647932558653966460912964485513216	32	64	32	64	2	4	2	4	16
42535295865117307932921825928971026432	32	64	32	64	2	4	2	4	16
85070591730234615865843651857942052864	32	64	32	64	2	4	2	4	16
170141183460469231731687303715884105728	32	64	32	64	2	4	2	4	16
340282366920938463463374607431768211456	32	64	32	64	2	4	2	4	16
680564733841876926926749214863536422912	32	64	32	64	2	4	2	4	16
1361129467683753853853498429727072845824	32	64	32	64	2	4	2	4	16
2722258935367507707706996859454145691648	32	64	32	64	2	4	2	4	16
5444517870735015415413993718908291383296	32	64	32	64	2	4	2	4	16
10889035741470030830827987437816582766592	32	64	32	64	2	4	2	4	16
21778071482940061661655974875633165533184	32	64	32	64	2	4	2	4	16
43556142965880123323311949751266331066368	32	64	32	64	2	4	2	4	16
87112285931760246646623899502532662132736	32	64	32	64	2	4	2	4	16
17422457186352049329324779900									

Table L6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1013

MEAN	C (OUT) / C (IN)		C (PIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
8	0.1903	0.0123	0.0996	0.1029	0.1009	0.1017	0.1030
16	0.1937	0.0122	0.1006	0.1053	0.1022	0.1037	0.0996
LEADTIME	0.1868	0.0123	0.0986	0.1005	0.0995	0.0997	
2	0.1901	0.0117	0.0992	0.1025			
4	0.1905	0.0129	0.1000	0.1033			
C (PIX) / C (IN)	0.1872	0.0120					
32	0.1933	0.0125					
64							
LEADTIME C (PIX) / C (IN)	0.1871	0.0113					
2	0.1930	0.0120					
4	0.1873	0.0127					
64	0.1936	0.0131					

Table L7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0132

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
	0.1603	0.0072	0.0127	0.0137	0.0116	0.0147	0.0172 0.0111
R	0.2099	0.0095	0.0162	0.0183	0.0151	0.0194	
16	0.1355	0.0061	0.0109	0.0114	0.0099	0.0124	
LEADTIME							
2	0.1452	0.0062	0.0111	0.0122			
4	0.1754	0.0082	0.0142	0.0152			
C (FIX) / C (IN)							
32	0.1519	0.0070					
64	0.1687	0.0074					
LEADTIME C (FIX) / C (IN)							
2	0.1370	0.0060					
4	0.1534	0.0064					
32	0.1668	0.0080					
64	0.1839	0.0084					

Table L8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM = 0.2833

C (OUT) /C (IN)	C (FIX) /C (IN)				LEADTIME				MEAN
	4	99	32	64	2	4	8	16	
0.2857	0.2808	0.3223	0.2442	0.2879	0.2786		0.2356	0.3309	
0.2381	0.2332	0.2694	0.2019	0.2382	0.2331				
0.3333	0.3284	0.3752	0.2866	0.3376	0.3241				
0.2903	0.2855	0.3286	0.2472						
0.2811	0.2761	0.3159	0.2413						

## LEADTIME

C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME		
		4	99	32	64	2	4
	0.2857	0.2808	0.3223	0.2442	0.2879	0.2786	
8	0.2381	0.2332	0.2694	0.2019	0.2382	0.2331	
16	0.3333	0.3284	0.3752	0.2866	0.3376	0.3241	

LEADTIME	C (FIX)/C (IN)	C (OUT)/C (IN)			C (FIX)/C (IN)		
		4	99	32	64	2	4
		0.2857	0.2808	0.3223	0.2442	0.2879	0.2786
8		0.2381	0.2332	0.2694	0.2019	0.2382	0.2331
16		0.3333	0.3284	0.3752	0.2866	0.3376	0.3241



Table L15

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			SOURCES OF TOTAL COST (% EXCESS OVER DP)			MEAN
	8.6	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN						
8		4 99	32 64	2 4	8 16	
16		4.8 10.9	9.3 7.9	7.5 9.5	9.2 8.1	
LEADTIME						
2	5.3 11.6		9.7 8.8	8.0 10.2		
4	4.4 10.4		9.1 7.3	7.1 9.0		
	4.0 9.8		7.9 7.2			
	5.6 11.9		10.6 8.6			
C (FIX)/C (IN)						
32	5.2 11.8					
64	4.4 10.2					
LEADTIME C (FIX)/C (IN)						
2	4.0 10.3					
4	3.9 9.4					
8	6.3 13.1					
16	5.0 10.9					









SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTPM=		0.7		BACKLOG FREQUENCY (% EXCESS OVER DP)		MEAN	
	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME				
4	99	32	64	2	4	8	16
-0.8	32.9	-0.7	2.1	0.6	0.8	3.0	-1.5
1.5	34.5	0.8	5.1	2.6	3.4		
-3.1	31.3	-2.2	-0.9	-1.4	-1.7		
-0.7	27.1	-1.0	2.2				
-1.0	38.6	-0.3	2.0				
-2.2	30.1						
0.5	35.7						
-2.2	23.6						
0.9	30.6						
-2.1	36.4						

Table L20

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL I  
 STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			WEIGHTED PROPORTION OF DEMAND BACKLOGGED (% EXCESS OVER DP)			MEAN
23.2						
MPAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	C (OUT)/C (IN)	C (FIX)/C (IN)	MEAN
8	4 99	32 64	2 4	2 4	8 16	
16	2.5 50.5	21.5 24.8	19.4 26.4	25.3 21.6		
LEADTIME						
2	5.6 50.6	20.0 30.5	20.4 29.5			
4	0.2 50.4	22.6 20.6	18.6 24.1			
C (FIX)/C (IN)						
32	2.1 41.9	16.8 21.7				
64	2.8 57.6	25.4 27.4				
LEADTIME						
2	1.2 47.4					
4	3.7 53.5					
C (FIX)/C (IN)						
32	0.1 38.1					
64	4.0 45.7					
LEADTIME						
2	2.0 55.1					
4	3.5 60.1					

Table L21

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

REPLENISHMENT FREQUENCY  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTPM=		-0.3															
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		LEADTIME		MEAN							
		4	99	32	64	2	4	2	4	8	16						
		6.5	-6.4	-0.5	-0.1	-0.7	0.2	-0.7	0.2	2.1	-1.9						
		8.9	-4.1	0.7	4.0	1.1	3.0	1.1	3.0								
		4.9	-7.9	-1.2	-2.8	-2.0	-1.8	-2.0	-1.8								
		5.9	-6.7	-0.5	-1.1												
		7.1	-6.0	-0.4	1.0												
		6.7	-6.8														
		6.3	-5.7														
		6.8	-6.9														
		4.8	-6.5														
		6.6	-6.7														
		7.7	-5.0														

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

2

4

4

## Appendix M

### Multi-item Forecasts for the (24,24) Statistical Power Approximation

#### Model I

Forecasting Properties of Inventory System of 16 Items with  
Negative Binomial Demand Distributions (Variance/Mean = 3)  
Controlled with Statistical Information, Revision Taking Place  
Every 24 Periods Using a 24-Period Demand History and Regression  
Estimates of Demand Means and Variances. Forecasts Made at Each  
Revision Using a 24-Period Demand History.

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Table M1 Sources of Forecast of Total Cost	M1
M2 Sources of Forecast of Aggregate Period-End Inventory	M2
M3 Sources of Forecast of Aggregate Backlog Cost	M3
M4 Sources of Forecast of Aggregate Replenishment Cost	M4
M5 Forecast of Backlog Frequency	M5
M6 Forecast of Weighted Proportion of Demand Backlogged	M6
M7 Forecast of Replenishment Frequency	M7
M8 Sources of Variance of Forecast of Total Cost	M8
M9 Sources of S.D. of Forecast of Total Cost	M9
M10 Sources of S.D. of Forecast of Aggregate Period-End Inventory	M10
M11 Sources of S.D. of Forecast of Aggregate Backlog Cost	M11
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M18 Backlog Frequency (% Underestimate of Actual by Forecast)	M18
M19 Weighted Proportion of Demand Backlogged (% Underestimate of Actual by Forecast)	M19
M20 Replenishment Frequency (% Underestimate of Actual by Forecast)	M20

Note: The corresponding appendices in MacCormick (1974), are his  
Appendices T and U.



Table M1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF TOTAL COST				MEAN	
OVERALL AGGREGATE FOR SYSTEM= 737.6					
	C (OUT)/C (IN)		C (FIX)/C (IN)	LEADTIME	
	4	99		2	4
MEAN					
8					8
16					16
LEADTIME					
2				48.0	52.0
4					
C (FIX)/C (IN)					
32					
64					
LEADTIME C (FIX)/C (IN)					
2					
2					
4					
4					

Table M2

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		454.0	PERCENT OF TOTAL COST=		61.6	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	
MEAN	4	99	32	64	2	4
	28.2	71.8	47.4	52.6	46.2	53.8
LEADTIME	8	11.6	30.3	19.9	22.1	19.4
	16	16.6	41.4	27.4	30.6	26.8
LEADTIME	2	13.2	33.0	21.7	24.6	22.5
	4	15.0	38.8	25.7	28.1	31.2
C (PIX)/C (IN)		13.0	34.3			
LEADTIME		15.2	37.4			
LEADTIME	2	6.0	15.6			
	2	7.2	17.4			
LEADTIME	4	7.0	18.7			
	4	8.0	20.1			
C (FIX)/C (IN)						
LEADTIME						
C (FIX)/C (IN)						
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LEADTIME						

Table M3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		74.7	PERCENT OF TOTAL COST=		10.1	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	4	99	32	64	2	4
	8					8
LEADTIME	16					16
	2	34.9	16.4		51.3	48.7
C (FIX)/C (IN)	4	39.5	9.2			
	32	34.6	11.8			
LEADTIME	64	39.8	13.8			
	2	16.5	7.8			
C (FIX)/C (IN)	4	18.5	8.6			
	32	18.1	4.0			
LEADTIME	64	21.4	5.2			
					42.3	57.7

Table M4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL I  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		208.9	PERCENT OF TOTAL COST= 28.3		MEAN	
	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		8	16
			4	99		
			50.4	49.6	39.8	60.2
					50.7	49.3
MEAN						
8			21.0	20.6	16.6	24.9
16			29.4	29.1	23.2	35.3
LEADTIME						
2			25.5	25.1	20.3	30.4
4			24.8	24.5	19.5	29.8
C (FIX)/C (IN)						
32			20.1	19.7		
64			30.3	29.9		
LEADTIME C (FIX)/C (IN)						
2			10.2	10.1		
2			15.3	15.1		
4			9.8	9.7		
4			15.0	14.8		
					41.5	58.5



Table M5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM = 0.0961

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
0.1871	0.0051	0.0941	0.0981	0.0980	0.0942	0.0969	0.0953
0.1891	0.0048	0.0946	0.0992	0.0990	0.0949		
0.1851	0.0055	0.0935	0.0970	0.0971	0.0934		
0.1898	0.0063	0.0972	0.0989				
0.1844	0.0040	0.0910	0.0973				
0.1838	0.0044						
0.1904	0.0059						

C (FIX)/C (IN)

32  
64

LEADTIME C (FIX)/C (IN)

2 32  
2 64  
4 32  
4 64

0.1891 0.0053  
0.1905 0.0073  
0.1785 0.0034  
0.1902 0.0045

Table M6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL I  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0076

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME			MEAN
		4	32	64	2	4	8	
8	0.1448 0.0020	0.0070	0.0081		0.0077	0.0074		0.0096 0.0065
16	0.1888 0.0023	0.0088	0.0104		0.0099	0.0093		
	0.1228 0.0018	0.0061	0.0070		0.0067	0.0064		
LEADTIME								
2	0.1359 0.0026	0.0073	0.0082					
4	0.1536 0.0014	0.0067	0.0080					
C (FIX)/C (IN)								
32	0.1346 0.0019							
64	0.1550 0.0022							
LEADTIME C (FIX)/C (IN)								
2	0.1281 0.0024							
4	0.1437 0.0027							
8	0.1410 0.0013							
16	0.1662 0.0016							

Table M7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

# REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2852

	C (OUT)/C (IN)	C (PIX)/C (IN)				LEADTIME				MEAN
	4	99	32	64	2	4	8	16		
MEAN	0.2873	0.2831	0.3247	0.2456	0.2894	0.2810	0.2373 0.3331			
P	0.2396	0.2349	0.2710	0.2035	0.2393	0.2353				
16	0.3349	0.3313	0.3785	0.2878	0.3396	0.3267				
LEADTIME										
2	0.2915	0.2873	0.3310	0.2479						
4	0.2831	0.2789	0.3185	0.2434						
C (FIX)/C (IN)										
32	0.3274	0.3221								
64	0.2472	0.2441								
LEADTIME C (FIX)/C (IN)										
2	0.3334	0.3285								
4	0.2496	0.2461								
8	0.3214	0.3156								
16	0.2448	0.2421								

Table M8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL I  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF VARIANCE OF TOTAL COST				
OVERALL AGGREGATE FOR SYSTEM= 1159.5				
	C (OUT) / C (IN)	C (FIX) / C (IN)	LEADTIME	MEAN
	4 99	32 64	2 4	8 16
	16.1 83.9	48.2 51.8	47.6 52.4	37.7 62.3
MEAN				
8	6.6 31.1	19.2 18.6	18.2 19.6	
16	9.5 52.8	29.0 33.2	29.5 32.8	
LEADTIME				
2	5.3 42.4	23.3 24.3		
4	10.8 41.5	24.9 27.5		
C (PIX) / C (IN)				
32	8.0 40.2			
64	8.1 43.7			
LEADTIME C (PIX) / C (IN)				
2	2.5 20.9			
4	2.8 21.5			
8	5.5 19.3			
16	5.3 22.2			



Table M9

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL I  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE POP SYSTEM=		SOURCES OF S.D. OF TOTAL COST			
		34.1			
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN
		4 99	32 64	2 4	8 16
MEAN		13.7 31.2	23.6 24.5	23.5 24.6	20.9 26.9
		8.8 19.0	14.9 14.7	14.5 15.1	
LEADTIME	8	10.5 24.7	18.4 19.6	18.5 19.5	
	16				
C (FIX)/C (IN)	2	7.8 22.2	16.5 16.8		
	4	11.2 21.9	17.0 17.9		
LEADTIME C (FIX)/C (IN)	32	9.6 21.6			
	64	9.7 22.5			
LEADTIME	2	5.4 15.6			
	4	5.7 15.8			
C (FIX)/C (IN)	32	8.0 15.0			
	64	7.8 16.0			



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(S,S) INVENTORY POLICIES IN A NONSTATIONARY DEMAND ENVIRONMENT.--ETC(U)  
APR 77 R L KAUFMAN.

DAAG29-76-G-0323

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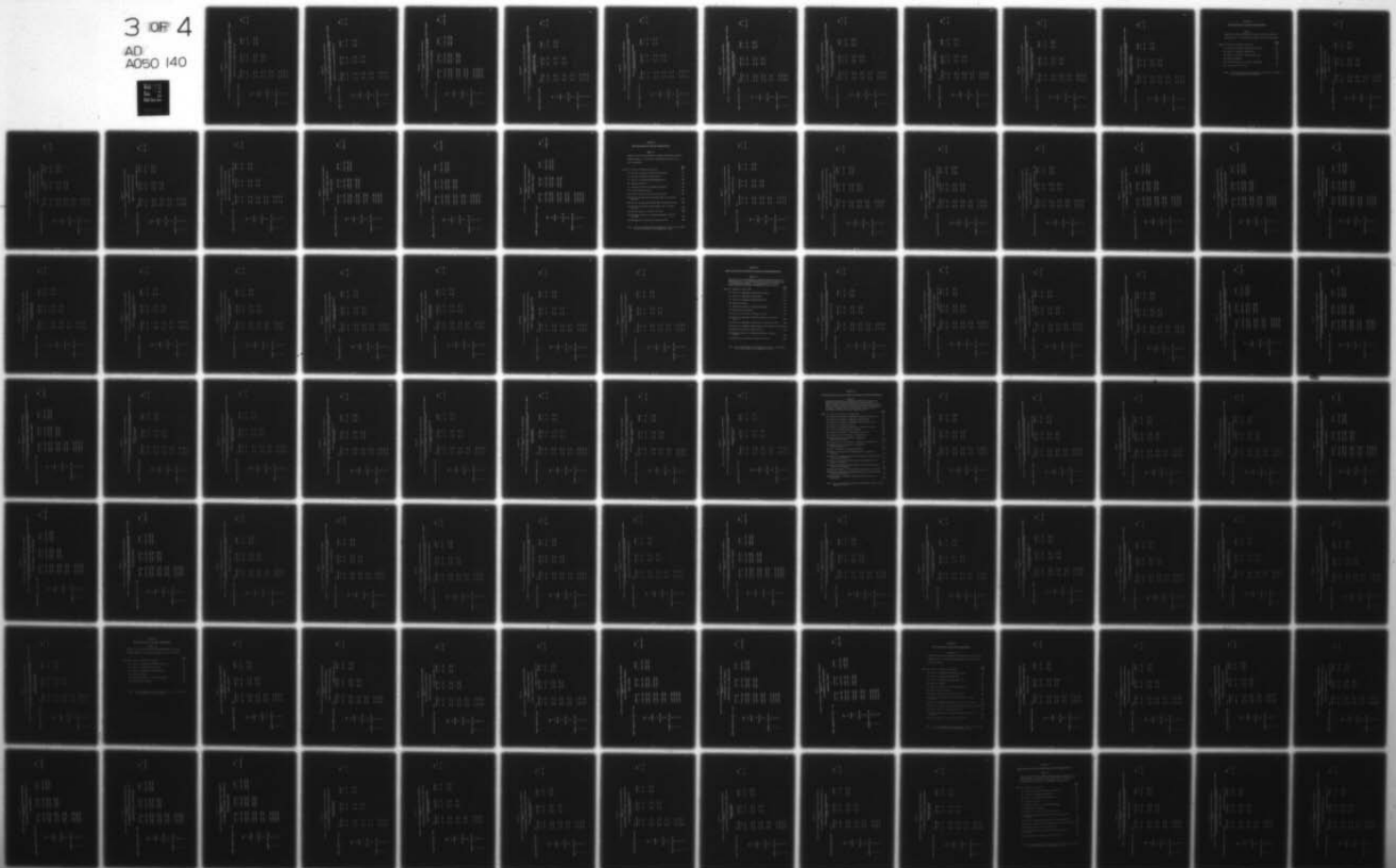
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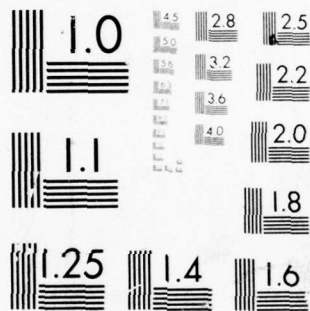
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MICROCOPY RESOLUTION TEST CHART  
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Table M11

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=				26.0	SOURCES OF S.D. OF AGGREGATE BACKLOG COST			
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN			
4	99	32	64	2	4	16		
10.2	23.9	18.5	18.3	19.7	17.0	15.1 21.2		
MEAN								
8	6.4	13.7	10.8	10.5	11.6	9.6		
16	8.0	19.6	15.0	15.0	15.9	14.0		
LEADTIME								
2	5.9	18.9	14.2	13.7				
4	8.4	14.7	11.8	12.1				
C (FIX)/C (IN)								
32	7.2	17.0						
64	7.2	16.8						
LEADTIME C (FIX)/C (IN)								
2	4.0	13.6						
2	4.3	13.0						
4	6.0	10.2						
4	5.8	10.7						



Table M13

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## S.D. OF BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM = 0.1727

	C (OUT) / C (IN)	C (FIX) / C (IN)	LEADTIME		MEAN
			2	4	
4	99	32 64			8 16
MEAN	0.1678 0.0407	0.1258 0.1183	0.1185 0.1256		0.1237 0.1205
8		0.0889 0.0860	0.0871 0.0878		
16		0.0891 0.0812	0.0803 0.0899		
LEADTIME					
2		0.0832 0.0843			
4		0.0944 0.0829			
C (FIX) / C (IN)					
32	0.1228 0.0274				
64	0.1144 0.0301				
LEADTIME C (FIX) / C (IN)					
2	0.0807 0.0205				
4	0.0811 0.0232				
8	0.0926 0.0182				
16	0.0807 0.0192				

Table M14

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL 1

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=		9.0		SOURCES OF TOTAL COST (% UNDERESTIMATE OF ACTUAL)		LEADTIME		MEAN	
		C (OUT)/C (IN)		C (FIX)/C (IN)					
	4	99		32	64	2	4	8	16
		4.5	11.7	10.0	8.2	6.8	11.0	9.6	8.6
MEAN									
	8	4.5	12.5	10.6	8.7	7.3	11.6		
		4.5	11.1	9.5	7.8	6.4	10.6		
LEADTIME									
	2	2.9	9.1	7.2	6.4				
		5.9	13.9	12.3	9.8				
C (FIX)/C (IN)									
	32	5.5	12.5						
		3.7	11.0						
LEADTIME C (FIX)/C (IN)									
	2	3.2	9.6						
		2.8	8.7						
	64								
	4	7.5	15.0						
		4.6	13.0						
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								
LEADTIME									
	32								
	64								
	4								
LEADTIME C (FIX)/C (IN)									
	2								
	64								
	4								



Table M15

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL I  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE PERIOD-END INVENTORY  
 (% UNDERESTIMATE OF ACTUAL)

OVERALL AGGREGATE FOR SYSTEM=		4.0											MEAN
LEADTIME	C (FIX) /C (IN)	C (OUT) /C (IN)		C (FIX) /C (IN)		LEADTIME							
		4	99	32	64	2	4	8	16				
		6.1	3.1	4.3	3.7	3.2	4.6	4.0	3.9				
MEAN													
8		5.8	3.3	4.8	3.4	3.5	4.4						
16		6.2	3.0	3.9	3.9	3.0	4.7						
LEADTIME													
2		4.2	2.8	3.3	3.2								
4		7.6	3.4	5.1	4.2								
C (FIX) /C (IN)													
32		6.8	3.3										
64		5.4	3.0										
LEADTIME C (FIX) /C (IN)													
2		4.4	2.9										
2		4.0	2.8										
4		8.7	3.6										
4		6.7	3.1										



Table M17

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST  
(% UNDERESTIMATE OF ACTUAL)

OVERALL AGGREGATE FOR SYSTEM=		-0.7		C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN	
				4 99		32 64		2 4		8 16	
				-0.5 -0.8		-0.8 -0.6		-0.4 -0.9		-0.7 -0.6	
MEAN											
8				-0.6 -0.8		-0.6 -0.8		-0.6 -0.9			
16				-0.5 -0.7		-0.9 -0.4		-0.4 -0.8			
LEADTIME											
2				-0.3 -0.6		-0.7 -0.3					
4				-0.8 -1.0		-0.8 -0.9					
C (FIX) / C (IN)											
32				-0.6 -0.9							
64				-0.5 -0.6							
LEADTIME C (FIX) / C (IN)											
2				-0.6 -0.8							
2				-0.2 -0.4							
4				-0.6 -1.1							
4				-0.9 -0.9							

Table M18

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL I  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			BACKLOG FREQUENCY (% UNDERESTIMATE OF ACTUAL)			MEAN
	5.1					
	C (OUT) / C (IN)		C (FIX) / C (IN)	LEADTIME		
	4 99		32 64	2 4		8 16
	1.7 58.2		5.6 4.7	2.8 7.4		5.9 4.3
MEAN						
8	2.4 60.9		6.0 5.8	3.2 8.5		
16	0.9 55.5		5.1 3.5	2.4 6.2		
LEADTIME						
2	0.1 46.0		2.0 3.5			
"	3.2 69.2		9.0 5.8			
C (FIX) / C (IN)						
32	1.8 63.6					
64	1.5 53.0					
LEADTIME C (FIX) / C (IN)						
2	-1.0 53.1					
2	1.3 39.2					
4	4.7 72.9					
4	1.7 65.7					





Table M20

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL I

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

REPLENISHMENT FREQUENCY  
(% UNDERESTIMATE OF ACTUAL)

OVERALL AGGREGATE FOR SYSTEM=		-0.7																	
		C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN											
		4	99	32	64	2	4	8	16										
		-0.6	-0.8	-0.8	-0.6	-0.5	-0.9	-0.7	-0.7										
		-0.7	-0.7	-0.6	-0.8	-0.4	-0.9	-0.4	-0.9										
		-0.5	-0.9	-0.9	-0.4	-0.7	-0.3	-0.7	-0.3										
		-0.4	-0.6	-0.7	-0.3	-0.7	-0.9	-0.7	-0.9										
		-0.6	-0.9	-0.6	-0.9	-0.6	-0.9	-0.6	-0.9										
		-0.5	-0.6	-0.5	-0.6	-0.5	-0.6	-0.5	-0.6										
		-0.6	-0.8	-0.6	-0.8	-0.6	-0.8	-0.6	-0.8										
		-0.2	-0.4	-0.2	-0.4	-0.2	-0.4	-0.2	-0.4										
		-0.6	-1.1	-0.6	-1.1	-0.6	-1.1	-0.6	-1.1										
		-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9										

## Appendix N

### Multi-item Data for the Power Approximation

#### Model II

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled Optimally with Full Information.

	<u>page</u>
Table N1 Sources of Expected Total Cost	N1
N3 Sources of Aggregate Period-End Inventory	N2
N4 Sources of Aggregate Backlog Cost	N3
N5 Sources of Aggregate Replenishment Cost	N4
N6 Backlog Frequency	N5
N7 Weighted Proportion of Demand Backlogged	N6
N8 Replenishment Frequency	N7

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.

Table N1  
 SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL II  
 FULL INFORMATION, OPTIMAL CONTROL (DP)

OVERALL AGGREGATE FOR SYSTEM=			751.8			SOURCES OF TOTAL COST		
		C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	MEAN		
MEAN	8	4 99	32 64	2 4	8 16			
	16	38.7 61.3	45.3 54.7	47.3 52.7	41.8 58.2			
LEADTIME	2	16.0 25.8	19.0 22.8	19.8 22.0				
	4	22.6 35.6	26.3 31.9	27.5 30.6				
C (FIX)/C (IN)	32	16.5 28.8	21.2 26.1					
	64	20.1 32.5	24.0 28.7					
LEADTIME	2	17.1 28.2						
	4	21.6 33.2						
C (FIX)/C (IN)	32	8.1 13.1						
	64	10.4 15.7						
LEADTIME	2	9.0 15.0						
	4	11.2 17.5						





Table N4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

## FULL INFORMATION, OPTIMAL CONTROL (DP)

## SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		106.4	PERCENT OF TOTAL COST=		14.2	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	8	4 99	32 64	2 4		8 16
	16	56.8 43.2	48.7 51.3	45.6 54.4		42.8 57.2
LEADTIME	2	23.9 18.8	21.0 21.8	19.5 23.3		
	4	32.9 24.4	27.8 29.5	26.1 31.1		
C (FIX)/C (IN)	32	25.9 19.7	22.1 23.4			
	64	30.9 23.5	26.6 27.8			
LEADTIME C (FIX)/C (IN)	2	27.3 21.4				
	4	29.5 21.8				
2	32	12.3 9.8				
	64	13.6 9.9				
4	32	15.0 11.6				
	64	15.9 11.9				

Table N5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		207.8	PERCENT OF TOTAL COST= 27.6		MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	
MEAN	8	4 59	32 64	2 4	8 16
	16	47.5 52.5	40.1 59.9	51.0 49.0	40.7 59.3
LEADTIME	2	19.5 21.3	16.4 24.3	20.8 20.0	
	4	28.0 31.2	23.7 35.5	30.2 29.1	
C (FIX)/C (IN)	32	24.3 26.7	20.5 30.5		
	64	23.2 25.9	19.6 29.4		
LEADTIME C (FIX)/C (IN)	2	19.1 21.0			
	4	28.4 31.5			
LEADTIME C (FIX)/C (IN)	2	9.8 10.7			
	4	14.5 16.0			
LEADTIME C (FIX)/C (IN)	2	9.3 10.4			
	4	13.9 15.5			

Table N6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, OPTIMAL CONTROL (DP)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1005

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
8	0.1918	0.0092	0.1005	0.1005	0.1002	0.1008	0.0998
16	0.1906	0.0091	0.0999	0.0998	0.0994	0.1003	0.0998
LEADTIME	0.1930	0.0093	0.1010	0.1013	0.1010	0.1013	0.0998
2	0.1913	0.0091	0.0997	0.1007	0.1002	0.1008	0.0998
4	0.1923	0.0093	0.1012	0.1004	0.1010	0.1013	0.0998
C (FIX) / C (IN)	0.1917	0.0092	0.1012	0.1004	0.1010	0.1013	0.0998
32	0.1918	0.0092	0.1012	0.1004	0.1010	0.1013	0.0998
64	0.1918	0.0092	0.1012	0.1004	0.1010	0.1013	0.0998

LEADTIME C (FIX) / C (IN)

2	0.1903	0.0092
2	0.1922	0.0091
4	0.1932	0.0092
4	0.1914	0.0093



Table N7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, OPTIMAL CONTROL (DP)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0108

C (OUT)/C (IN)		C (FIX)/C (IN)				LEADTIME				MEAN
4	99	32	64	2	4	8	16			
	0.1575	0.0048	0.0105	0.0110	0.0098	0.0117	0.0138	0.0092		
8	0.1991	0.0063	0.0135	0.0141	0.0126	0.0151				
16	0.1367	0.0041	0.0090	0.0095	0.0084	0.0100				
LEADTIME										
2	0.1436	0.0044	0.0095	0.0101						
4	0.1714	0.0053	0.0114	0.0120						
C (FIX)/C (IN)										
32	0.1514	0.0048								
64	0.1635	0.0049								
LEADTIME										
2	0.1365	0.0044								
2	0.1506	0.0044								
4	0.1664	0.0052								
4	0.1765	0.0053								



## Appendix O

### Multi-item Data for the Power Approximation

#### Model II

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled Approximately Optimally with  
Full Information.

	<u>page</u>
Table 01 Sources of Expected Total Cost	01
03 Sources of Aggregate Period-End Inventory	02
04 Sources of Aggregate Backlog Cost	03
05 Sources of Aggregate Replenishment Cost	04
06 Backlog Frequency	05
07 Weighted Proportion of Demand Backlogged	06
08 Replenishment Frequency	07
015 Sources of Total Cost (% Excess Over DP)	08
016 Sources of Aggregate Period-End Inventory (% Excess Over DP)	09
017 Sources of Aggregate Backlog Cost (% Excess Over DP)	010
018 Sources of Aggregate Replenishment Cost (% Excess Over DP)	011
019 Backlog Frequency (% Excess Over DP)	012
020 Weighted Proportion of Demand Backlogged (% Excess Over DP)	013
021 Replenishment Frequency (% Excess Over DP)	014

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Table 01

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

OVERALL AGGREGATE FOR SYSTEM=		757.7		SOURCES OF TOTAL COST		MEAN	
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME	
		4	99	32	64	2	4
MEAN	8	38.6	61.4	45.2	54.8	47.3	52.7
	16	16.1	25.8	19.0	22.9	19.8	22.1
LEADTIME	2	22.5	35.6	26.2	31.9	27.5	30.6
	4	18.5	28.8	21.2	26.1		
C (FIX)/C (IN)	32	20.1	32.6	24.0	28.7		
	64	17.0	28.2				
LEADTIME C (FIX)/C (IN)	2	21.6	33.2				
	4	8.1	13.1				
	2	10.4	15.7				
	4	9.0	15.0				
	4	11.2	17.5				
						41.9	58.1



Table 03

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		442.5	PERCENT OF TOTAL COST=		58.4	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
NEAN	4	59	32	64	2	4
	8					16
LEADTIME	28.4	71.6	47.4	52.6	46.1	53.9
	11.5	30.2	19.7	22.0	19.1	22.6
	16.9	41.4	27.7	30.6	26.9	31.3
	13.3	32.8	21.6	24.5		
	15.2	38.8	25.8	28.1		
	13.2	34.2				
	15.3	37.3				
	6.1	15.5				
	7.2	17.3				
	7.1	18.7				
	8.1	20.0				

NEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64

41.7 56.3

Table 04

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		106.2	PERCENT OF TOTAL COST=		14.0	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
	4	99	32	64	2	4
					8	16
					43.4	56.6

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (PIX)/C (IN)

2

32

2

64

4

32

4

64

12.4

9.3

14.2

10.1

14.8

10.0

16.8

12.4

20.3

23.1

26.2

30.4

21.7

24.3

24.8

29.2

20.4

23.0

25.6

31.0

46.0

54.0

Table 05

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		206.9	PERCENT OF TOTAL COST=		27.6	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	8	4 99	32 64	2 4		8 16
	16	50.2 49.8	35.8 60.2	50.6 49.4		41.5 58.5
LEADTIME	2	20.8 20.7	16.7 24.8	21.0 20.5		
	4	29.4 29.1	23.1 35.4	29.6 28.9		
C (FIX)/C (IN)	32	25.4 25.2	20.2 30.4			
	64	24.8 24.6	19.6 29.8			
LEADTIME C (FIX)/C (IN)	2	20.0 19.8				
	4	30.2 30.0				
2	32	10.2 10.0				
	64	15.3 15.2				
4	32	9.9 9.7				
	64	14.9 14.8				

Table O6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

PULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1028

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
8	0.1969	0.0088	0.1005	0.1051	0.1029	0.1028	0.1053
16	0.2021	0.0085	0.1037	0.1070	0.1068	0.1039	0.1003
LEADTIME	0.1916	0.0090	0.0974	0.1033	0.0990	0.1017	
2	0.1969	0.0089	0.1007	0.1051			
4	0.1969	0.0087	0.1004	0.1052			

C (FIX)/C (IN)

32	0.1928	0.0083
64	0.2010	0.0093

LEADTIME C (FIX)/C (IN)

2	0.1927	0.0086
4	0.2010	0.0091
32	0.1928	0.0079
64	0.2010	0.0094



Table 07

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0107

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		MEAN
			2	4	
4	99	32 64	2	4	8 16
0.1611	0.0047	0.0100 0.0115	0.0099	0.0116	0.0140 0.0091
8	0.2127	0.0060	0.0131	0.0149	0.0132 0.0148
16	0.1353	0.0040	0.0084	0.0098	0.0082 0.0100
LPADTIME					
2	0.1471	0.0043	0.0093	0.0105	
4	0.1751	0.0050	0.0107	0.0125	
C (FIX)/C (IN)					
32	0.1506	0.0043			
64	0.1716	0.0050			
LEADTIME C (FIX)/C (IN)					
2	0.1370	0.0041			
4	0.1572	0.0045			
8	0.1642	0.0045			
16	0.1859	0.0055			

Table 08

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

PULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2853

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
8	0.2866	0.2839	0.3249	0.2456	0.2891	0.2814	0.2374
16	0.2385	0.2363	0.2722	0.2026	0.2410	0.2339	0.3331
LEADTIME	0.3348	0.3314	0.3775	0.2886	0.3373	0.3288	
2	0.2906	0.2877	0.3259	0.2484			
4	0.2827	0.2801	0.3198	0.2429			
C (FIX)/C (IN)	0.3268	0.3229					
32	0.2465	0.2448					
64							
LEADTIME	0.3320	0.3279					
2	0.2493	0.2475					
4	0.3217	0.3180					
4	0.2436	0.2422					



Table 016

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE PERIOD-END INVENTORY  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		1.1															
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME											
		4	99	32	64	2	4	8	16								
		-4.3	3.5	2.3	0.1	1.3	1.0	0.2	1.8								
		-6.8	3.2	1.2	-0.6	-0.4	0.7										
		-2.6	3.7	3.1	0.6	2.4	1.3										
		-4.1	3.6	2.0	0.6												
		-4.6	3.4	2.6	-0.4												
		-2.3	4.3														
		-6.0	2.8														
		-2.3	3.8														
		-5.5	3.4														
		-2.3	4.7														
		-6.5	2.3														

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64















## Appendix P

### Multi-item Data for the (24,24) Statistical Power Approximation

#### Model II

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled with Statistical Information from  
a 24-Period Demand History, with Revision Every 24 Periods,  
Using Regression Estimates of Demand Means and Variances.

	<u>page</u>
Table P1 Sources of Total Cost	P1
P3 Sources of Aggregate Period-End Inventory	P2
P4 Sources of Aggregate Backlog Cost	P3
P5 Sources of Aggregate Replenishment Cost	P4
P6 Backlog Frequency	P5
P7 Weighted Proportion of Demand Backlogged	P6
P8 Replenishment Frequency	P7
P15 Sources of Total Cost (% Excess Over DP)	P8
P16 Sources of Aggregate Period-End Inventory (% Excess Over DP)	P9
P17 Sources of Aggregate Backlog Cost (% Excess Over DP)	P10
P18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	P11
P19 Backlog Frequency (% Excess Over DP)	P12
P20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	P13
P21 Replenishment Frequency (% Excess Over DP)	P14

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices K and L.



Table P3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		469.0	PERCENT OF TOTAL COST=		57.8	MEAN
		C(OUT)/C(IN)	C(FIX)/C(IN)		LEADTIME	
		4 99	32 64	2 4		8 16
		28.9 71.1	47.7 52.3	45.8 54.2		42.1 57.9
MEAN						
8		11.9 30.2	20.1 22.0	19.3 22.8		
16		17.0 40.9	27.6 30.3	26.6 31.3		
LEADTIME						
2		13.3 32.5	21.6 24.2			
4		15.6 38.6	26.1 28.1			
C(FIX)/C(IN)						
32		13.5 34.2				
64		15.4 36.9				
LEADTIME C(FIX)/C(IN)						
2	32	6.1 15.5				
2	64	7.2 17.0				
4	32	7.4 18.7				
4	64	8.2 19.9				



Table P4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		133.0	PERCENT OF TOTAL COST=		16.4	MEAN
		C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME		
		4 99	32 64	2 4		8 16
		46.9 53.1	47.4 52.6	43.1 56.9		43.0 57.0
MEAN						
8		20.2 22.8	20.1 22.9	18.7 24.3		
16		26.7 30.3	27.2 29.8	24.4 32.6		
LEADTIME						
2		20.8 22.2	20.6 22.4			
4		26.1 30.9	26.7 30.2			
C(FIX)/C(IN)						
32		21.9 25.4				
64		25.0 27.7				
LEADTIME C(FIX)/C(IN)						
2		9.7 10.9				
4		11.1 11.3				
8		12.2 14.5				
16		13.8 16.4				

Table P5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST				PERCENT OF TOTAL COST= 25.8		MEAN
OVERALL AGGREGATE FOR SYSTEM=	209.2	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
	4	99	32	64	2	4
	50.3	49.7	39.8	60.2	50.6	49.4
MEAN						8
8	20.9	20.6	16.6	24.9	21.0	20.5
16	29.4	29.1	23.2	35.3	29.7	28.8
LEADTIME						
2	25.5	25.1	20.2	30.4		
4	24.8	24.6	19.6	29.8		
C (FIX)/C (IN)						
32	20.1	19.7				
64	30.2	30.0				
LEADTIME C (FIX)/C (IN)						
2	10.2	10.0				
4	15.3	15.1				
8	9.9	9.7				
16	14.9	14.8				
						41.5
						58.5

Table P6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL II  
 STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 0.1020				BACKLOG FREQUENCY			
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		MEAN	
		4 99	32 64	2 4	8 16		
		0.1916 0.0125	0.0994 0.1047	0.1003 0.1037	0.1029 0.1012		
		0.1935 0.0124	0.0997 0.1061	0.1017 0.1041			
		0.1897 0.0127	0.0991 0.1032	0.0990 0.1034			
		0.1890 0.0117	0.0978 0.1029				
		0.1941 0.0133	0.1011 0.1064				
		0.1867 0.0121					
		0.1964 0.0129					
		0.1840 0.0116					
		0.1940 0.0118					
		0.1894 0.0127					
		0.1989 0.0140					

Table P7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL 11

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0134

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
0.1625	0.0074		0.0127	0.0142	0.0116	0.0153	0.0174
0.2099	0.0096		0.0163	0.0185	0.0151	0.0196	
0.1388	0.0064		0.0110	0.0120	0.0098	0.0132	
0.1443	0.0062		0.0111	0.0121			
0.1806	0.0086		0.0144	0.0162			
0.1520	0.0071						
0.1730	0.0077						
0.1343	0.0061						
0.1543	0.0063						
0.1696	0.0081						
0.1917	0.0092						



Table P8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

# REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2856

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
8	0.2875	0.2837	0.3252	0.2460	0.2894	0.2818	0.2375
16	0.2391	0.2359	0.2716	0.2034	0.2402	0.2348	0.3337
LEADTIME	0.3359	0.3315	0.3787	0.2886	0.3386	0.3287	
2	0.2916	0.2872	0.3300	0.2487			
4	0.2834	0.2801	0.3203	0.2432			

C (FIX) / C (IN)

32	0.3281	0.3223
64	0.2469	0.2450

LEADTIME C (FIX) / C (IN)

2	0.3331	0.3270
2	0.2500	0.2475
4	0.3230	0.3176
4	0.2439	0.2426



Table P16

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=				SOURCES OF AGGREGATE PERIOD-END INVENTORY (% EXCESS OVER DP)			
7.2							
	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN			
4	99	32 64	2 4	8	16		
3.0	9.0	9.1 5.5	6.7 7.6	7.1	7.2		
MEAN							
8	2.0 9.3	9.4 5.2	6.2 7.9				
16	3.8 8.7	8.9 5.7	7.1 7.3				
LEADTIME							
2	2.2 8.7	8.5 5.2					
4	3.8 9.2	9.7 5.7					
C (PIX)/C (IN)							
32	6.2 10.3						
64	0.4 7.7						
LEADTIME C (PIX)/C (IN)							
2	5.0 9.9						
2	-0.1 7.7						
4	7.3 10.6						
4	0.8 7.8						





Table P18

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=										0.7	
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN			
		4	99	32	64	2	4	8	16		
		6.7	-4.8	-0.2	1.2	0.0	1.3	2.5	-0.6		
MEAN		8		2.0	2.9	1.7	3.4				
		16		-1.6	0.0	-1.1	-0.1				
LEADTIME		2		-0.7	0.5						
		4		0.5	1.9						
C(FIX)/C(IN)		32		5.9	-5.6						
		64		7.2	-4.2						
LEADTIME		2		4.6	-5.7						
		2		6.2	-4.7						
		4		7.3	-5.6						
		4		8.1	-3.6						





Table P21

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

REPLENISHMENT FREQUENCY  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=										0.4	
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		LEADTIME		MEAN	
		4	99	32	64	2	4	2	4	8	16
		6.4	-5.0	-0.2	1.2	-0.2	1.1	-0.2	1.1	2.4	-0.9
		8.1	-2.8	2.0	2.9	1.4	3.4				
		5.3	-6.5	-1.5	0.0	-1.3	-0.5				
		5.3	-5.2	-0.7	0.5						
		7.6	-4.8	0.5	1.9						
		5.9	-5.6								
		7.2	-4.2								
		4.6	-5.7								
		6.2	-4.7								
		7.3	-5.6								
		8.1	-3.6								

		C (PIX)/C (IN)	
		32	64
		5.9	-5.6
		7.2	-4.2
		4.6	-5.7
		6.2	-4.7
		7.3	-5.6
		8.1	-3.6

		C (PIX)/C (IN)	
		32	64
		5.9	-5.6
		7.2	-4.2
		4.6	-5.7
		6.2	-4.7
		7.3	-5.6
		8.1	-3.6

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64



## Appendix Q

### Multi-item Forecasts for the (24,24) Statistical Power Approximation

#### Model II

Forecasting Properties of Inventory System of 16 Items with  
Negative Binomial Demand Distributions (Variance/Mean = 3)  
Controlled with Statistical Information, Revision Taking Place  
Every 24 Periods Using a 24-Period Demand History and Regression  
Estimates of Demand Means and Variances. Forecasts Made at  
Each Revision Using a 24-Period Demand History.

	<u>page</u>
Table Q1 Sources of Forecast of Total Cost	Q1
Q2 Sources of Forecast of Aggregate Period-End Inventory	Q2
Q3 Sources of Forecast of Aggregate Backlog Cost	Q3
Q4 Sources of Forecast of Aggregate Replenishment Cost	Q4
Q5 Forecast of Backlog Frequency	Q5
Q6 Forecast of Weighted Proportion of Demand Backlogged	Q6
Q7 Forecast of Replenishment Frequency	Q7
Q8 Sources of Variance of Forecast of Total Cost	Q8
Q9 Sources of S.D. of Forecast of Total Cost	Q9
Q10 Sources of S.D. of Forecast of Aggregate Period-End Inventory	Q10
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Q13 S.D. of Forecast of Backlog Frequency	Q13
Q14 Sources of Total Cost (% Underestimate of Actual by Forecast)	Q14
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Q16 Sources of Aggregate Backlog Cost (% Underestimate of Actual by Forecast)	Q16
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Q19 Weighted Proportion of Demand Backlogged (% Underestimate of Actual by Forecast)	Q19
Q20 Replenishment Frequency (% Underestimate of Actual by Forecast)	Q20

Note: The corresponding appendices in MacCormick (1974), are his  
Appendices T and U.

Table Q1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL II  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 737.8				SOURCES OF TOTAL COST			
	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN			
					2	4	8
MEAN							
8	16.3	25.5	19.0	22.9	20.1	21.8	
16	22.9	35.3	26.1	32.1	27.8	30.4	
LEADTIME							
2	19.0	28.9	21.4	26.5			
4	20.2	31.9	23.7	28.4			
C (FIX)/C (IN)							
32	17.2	27.9					
64	22.0	32.9					
LEADTIME C (FIX)/C (IN)							
2	8.3	13.1					
4	10.7	15.8					
8	8.9	14.8					
16	11.3	17.1					

41.8 58.2

Table Q2

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		452.3	PERCENT OF TOTAL COST=		61.3	MEAN
C (OUT)/C (IN)			C (FIX)/C (IN)		LEADTIME	
4	99		32	64	2	4
		28.1	71.9	47.5	52.5	46.5
						53.5
						42.0
						58.0
MEAN						
8		11.5	30.4	20.0	22.0	19.5
						22.4
16		16.6	41.4	27.5	30.5	26.9
						31.1
LEADTIME						
2		13.3	33.2	21.8	24.6	
4		14.9	38.7	25.7	27.9	
C (FIX)/C (IN)						
32		13.0	34.5			
64		15.1	37.4			
LEADTIME C (FIX)/C (IN)						
2	32	6.1	15.8			
2	64	7.2	17.4			
4	32	7.0	18.7			
4	64	7.9	20.0			

Table Q3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM		76.2	PERCENT OF TOTAL COST= 10.3		MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	
MEAN	8	4 99	32 64	2 4	8 16
	16	74.3 25.7	45.0 55.0	48.4 51.6	41.7 58.3
LEADTIME	2	32.1 9.6	19.3 22.5	20.6 21.2	
	4	42.2 16.1	25.7 32.6	27.8 30.4	
C (FIX)/C (IN)	32	34.8 13.7	21.6 26.8		
	64	39.5 12.0	23.4 26.2		
LEADTIME C (FIX)/C (IN)	2	34.2 10.8			
	4	40.1 14.9			
LEADTIME C (FIX)/C (IN)	2	16.1 5.5			
	4	18.6 8.2			
LEADTIME C (FIX)/C (IN)	2	18.0 5.3			
	4	21.5 6.7			



Table Q4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL II  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST				
OVERALL AGGREGATE FOR SYSTEM=		209.3	PERCENT OF TOTAL COST= 28.4	
C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME
4 99		32 64	2 4	MEAN
50.3 49.7		39.8 60.2	50.7 49.3	8 16
20.9 20.7		16.7 24.9	21.0 20.6	41.6 58.4
29.4 29.1		23.2 35.2	29.7 28.8	
25.5 25.2		20.2 30.4		
24.8 24.6		19.6 29.7		
20.1 19.8				
30.2 29.9				
10.2 10.1				
15.3 15.1				
9.9 9.7				
14.9 14.8				

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64



Table Q6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL II  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0077

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME			MEAN
		4	32	64	2	4	8	
0.1475	0.0021	0.0069	0.0085	0.0075	0.0080			
0.1914	0.0023	0.0089	0.0104	0.0095	0.0098			
0.1255	0.0019	0.0059	0.0075	0.0064	0.0070			
0.1380	0.0022	0.0067	0.0083					
0.1569	0.0019	0.0072	0.0087					
0.1356	0.0017							
0.1593	0.0024							
0.1281	0.0018							
0.1479	0.0026							
0.1431	0.0017							
0.1708	0.0022							

Table Q7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL II  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

REPLENISHMENT FREQUENCY				MEAN			
OVERALL AGGREGATE FOR SYSTEM= 0.2858							
	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME				
	4 99	32 64	2 4			8 16	
MEAN	0.2875 0.2841	0.3256 0.2460	0.2899 0.2816			0.2380 0.3335	
8	0.2394 0.2367	0.2723 0.2037	0.2410 0.2350				
16	0.3357 0.3314	0.3789 0.2882	0.3388 0.3282				
LEADTIME							
2	0.2920 0.2879	0.3310 0.2489					
4	0.2831 0.2802	0.3203 0.2430					
C (FIX)/C (IN)							
32	0.3280 0.3233						
64	0.2471 0.2448						
LEADTIME C (FIX)/C (IN)							
2	0.3332 0.3287						
4	0.2507 0.2471						
8	0.3227 0.3178						
16	0.2434 0.2426						



Table Q8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF VARIANCE OF TOTAL COST				MEAN	
OVERALL AGGREGATE FOR SYSTEM= 1432.9					
	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN					
8				2	4
16				8	16
LEADTIME					
2	4	32	64	38.2	61.8
4	13.6	86.4	47.6	52.4	33.5
					66.5
	5.5	28.0	15.9	17.6	
	8.0	58.4	31.7	34.7	
	4.6	33.7	16.6	21.6	
	9.0	52.7	31.0	30.7	
C (PIX)/C (IN)					
32	6.5	41.1			
64	7.1	45.3			
LEADTIME C (FIX)/C (IN)					
2	2.1	14.5			
2	2.4	19.2			
4	4.4	26.6			
4	4.6	26.1			

Table Q9

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF S.D. OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM=		37.9	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN	
			4	99	32	64	2	4	8	16
			13.9	35.2	26.1	27.4	23.4	29.7	21.9	30.9
MEAN										
	8		8.9	20.0	15.1	15.9	15.0	16.0		
	16		10.7	28.9	21.3	22.3	18.0	25.1		
LEADTIME										
	2		8.1	22.0	15.4	17.6				
	4		11.4	27.5	21.1	21.0				
C (FIX)/C (IN)										
	32		9.7	24.3						
	64		10.1	25.5						
LEADTIME C (FIX)/C (IN)										
	2		5.5	14.4						
	2		5.9	16.6						
	4		7.9	19.5						
	4		8.1	19.3						



Table Q11

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF S.D. OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM= 29.9

C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN	
4	99	32	64	2	4	8	16
10.5	28.0	20.4	21.9	18.9	23.1	16.0	25.3

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64

4.0 11.6

4.8 13.6

5.9 15.1

6.1 15.3

7.1 19.1

7.8 20.4

6.2 17.9

8.5 21.5

6.5 14.6

8.3 23.9

10.8 11.8

17.3 18.4

12.3 14.4

16.3 16.5

11.9 10.6

14.7 20.5



Table Q12

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF S.D. OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		5.4		C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN	
	4	99	32	64	2	4	8	16			
	3.8	3.8	2.9	4.5	3.9	3.7	3.7	3.9			
MEAN											
	8	2.6	2.6	2.0	3.2	2.6	2.7				
	16	2.8	2.7	2.2	3.2	2.9	2.6				
LEADTIME											
	2	2.7	2.7	2.1	3.2						
	4	2.7	2.6	2.0	3.2						
C (FIX) / C (IN)											
	32	2.1	2.1								
	64	3.2	3.2								
LEADTIME C (FIX) / C (IN)											
	2	1.5	1.5								
	2	2.3	2.3								
	4	1.4	1.4								
	4	2.3	2.2								

Table Q13

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

S.D. OF BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTPM= 0.1729

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
	0.1679	0.0413	0.1194	0.1251	0.1189	0.1256	0.1279 0.1164
8	0.1249	0.0276	0.0862	0.0946	0.0890	0.0919	
16	0.1122	0.0307	0.0827	0.0819	0.0789	0.0856	
LEADTIME	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	2	4	32	64	2	4	
	0.1151	0.0300	0.0817	0.0864	0.0890	0.0919	
	0.1223	0.0285	0.0871	0.0904	0.0789	0.0856	
LEADTIME C (FIX) / C (IN)	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	2	4	32	64	2	4	
	0.1162	0.0275	0.0817	0.0864	0.0890	0.0919	
	0.1212	0.0309	0.0871	0.0904	0.0789	0.0856	
LEADTIME C (FIX) / C (IN)	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	2	4	32	64	2	4	
	0.0792	0.0199	0.0817	0.0864	0.0890	0.0919	
	0.0835	0.0224	0.0871	0.0904	0.0789	0.0856	
	0.0850	0.0190	0.0817	0.0864	0.0890	0.0919	
	0.0879	0.0212	0.0871	0.0904	0.0789	0.0856	









Table Q17

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL II  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST  
 (% UNDERESTIMATE OF ACTUAL)

OVERALL AGGREGATE FOR SYSTEM=		-0.0																										
		C (OUT)/C (IN)			C (FIX)/C (IN)			LEADTIME			MEAN																	
		4	99		32	64		2	4		8	16																
		-0.0	-0.1		-0.1	0.0		-0.2	0.1		-0.2	0.1																
		-0.2	-0.2		-0.3	-0.2		-0.3	-0.2		-0.3	-0.2																
		0.1	0.1		-0.0	0.1		-0.1	0.2		-0.1	0.2																
		-0.2	-0.1		-0.3	-0.1		-0.3	-0.1		-0.3	-0.1																
		0.1	-0.0		0.0	0.1		0.0	0.1		0.0	0.1																
		0.0	-0.3																									
		-0.1	0.1																									
		-0.0	-0.5																									
		-0.3	0.2																									
		0.1	-0.1																									
		0.2	0.0																									









## Appendix R

### Multi-item Data for the Power Approximation

#### Model III

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Optimally with Full Information.

	<u>page</u>
Table R1 Sources of Expected Total Cost	R1
R3 Sources of Aggregate Period-End Inventory	R2
R4 Sources of Aggregate Backlog Cost	R3
R5 Sources of Aggregate Replenishment Cost	R4
R6 Backlog Frequency	R5
R7 Weighted Proportion of Demand Backlogged	R6
R8 Replenishment Frequency	R7

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.



Table R3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		428.9	PERCENT OF TOTAL COST= 59.7		MEAN	
LEADTIME	C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	8	16
2	32	4	32	2	4	42.5
4	64	30.4	46.6	45.5	54.5	57.5
8	128	12.7	29.8	19.8	22.7	19.4
16	256	17.7	39.8	26.8	30.7	23.1
32	512	14.0	31.5	21.0	24.5	26.1
64	1024	16.4	38.1	25.6	29.0	31.4
128	2048	13.8	32.8			
256	4096	16.6	36.8			
512	8192	6.3	14.7			
1024	16384	7.7	16.7			
2048	32768	7.5	10.0			
4096	65536	8.9	20.1			



Table R4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		101.6	PERCENT OF TOTAL COST=		14.1	MEAN
		C (OUT) / C (IN)	C (FIX) / C (IN)		LEADTIME	
MEAN	8	4 99	32 64	2 4	8 16	43.5 56.5
	16	56.5 43.5	49.2 50.8	45.1 54.9		
LEADTIME	2	24.1 19.3	21.3 22.2	19.6 23.8		
	4	32.3 24.2	27.9 28.7	25.5 31.1		
C (FIX) / C (IN)	32	25.4 19.7	22.1 23.0			
	64	31.1 23.8	27.0 27.8			
LEADTIME C (FIX) / C (IN)	2	27.3 21.9				
	4	29.2 21.7				
LEADTIME C (FIX) / C (IN)	2	12.2 9.9				
	4	13.2 9.8				
LEADTIME C (FIX) / C (IN)	2	15.1 11.9				
	4	16.0 11.9				

Table R5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		187.7	PERCENT OF TOTAL COST= 26.1		MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	
MEAN	4	99	32	64	2
	8				4
LEADTIME	48.2	51.8	41.2	58.8	52.2
					47.8
C (FIX)/C (IN)	19.6	20.7	16.7	23.6	21.1
	28.6	31.1	24.5	35.2	31.1
LEADTIME	25.1	27.1	21.5	30.7	28.6
	23.1	24.7	19.7	28.1	
C (FIX)/C (IN)	19.6	21.6			
	28.6	30.3			
LEADTIME	10.3	11.2			
	14.8	15.9			
C (FIX)/C (IN)	9.3	10.3			
	13.8	14.4			

40.3 59.7

Table R6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, OPTIMAL CONTROL (DP)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1002

MEAN	C(OUT)/C(IN)	C(PIX)/C(IN)	LEADTIME		MEAN
			2	4	
8	0.1913 0.0092	0.0999 0.1006	0.0996 0.1009		0.0994 0.1011
16	0.1896 0.0091	0.0990 0.0998	0.0983 0.1004		
	0.1929 0.0093	0.1008 0.1014	0.1008 0.1014		
LEADTIME	C(PIX)/C(IN)	C(OUT)/C(IN)	LEADTIME		MEAN
			2	4	
2	0.1900 0.0091	0.0991 0.1000			
4	0.1926 0.0093	0.1007 0.1011			
C(PIX)/C(IN)	LEADTIME	C(OUT)/C(IN)	LEADTIME		MEAN
			2	4	
32	0.1905 0.0093	0.0991 0.1000			
64	0.1921 0.0091	0.1007 0.1011			
LEADTIME	C(PIX)/C(IN)	C(OUT)/C(IN)	LEADTIME		MEAN
			2	4	
2	0.1890 0.0092	0.0991 0.1000			
4	0.1911 0.0090	0.1007 0.1011			
8	0.1921 0.0094	0.1008 0.1014			
16	0.1930 0.0092	0.1009 0.1009			

Table R7  
 SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL III

FULL INFORMATION, OPTIMAL CONTROL (DP)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0103

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	8 16
0.1493	0.0047	0.0101	0.0104	0.0093	0.0113		0.0134 0.0087
0.1913	0.0062	0.0131	0.0137	0.0121	0.0147		
0.1283	0.0039	0.0086	0.0088	0.0078	0.0096		
LEADTIME							
2	0.1343	0.0042	0.0091	0.0094			
4	0.1643	0.0051	0.0111	0.0114			
C (FIX) / C (IN)							
32	0.1443	0.0047					
64	0.1543	0.0046					
LEADTIME C (FIX) / C (IN)							
2	0.1290	0.0042					
2	0.1396	0.0042					
4	0.1597	0.0051					
4	0.1690	0.0051					



Table R8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, OPTIMAL CONTROL (DP)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2587

MEAN	C (OUT)/C (IN)			C (FIX)/C (IN)			LEADTIME			MEAN
	4	99		32	64		2	4		
8	0.2483	0.2692		0.3018	0.2157		0.2702	0.2473		0.2088
16	0.2037	0.2140		0.2446	0.1731		0.2188	0.1988		0.3087
	0.2929	0.3244		0.3591	0.2582		0.3215	0.2958		

LEADTIME

C (FIX)/C (IN)	C (OUT)/C (IN)			C (FIX)/C (IN)			LEADTIME			MEAN
	4	99		32	64		2	4		
2	0.2592	0.2811		0.3153	0.2250		0.2702	0.2473		0.2088
4	0.2374	0.2572		0.2883	0.2063		0.2188	0.1988		0.3087

LEADTIME	C (FIX)/C (IN)			C (OUT)/C (IN)			C (FIX)/C (IN)			MEAN
	4	99		32	64		32	64		
2	0.3011	0.3295		0.2872	0.3164		0.3011	0.3295		0.2088
4	0.2172	0.2327		0.2094	0.2219		0.2172	0.2327		0.3087

## Appendix S

### Multi-item Data for the Power Approximation

#### Model III

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled Approximately Optimally with  
Full Information.

	<u>page</u>
Table S1 Sources of Expected Total Cost	S1
S3 Sources of Aggregate Period-End Inventory	S2
S4 Sources of Aggregate Backlog Cost	S3
S5 Sources of Aggregate Replenishment Cost	S4
S6 Backlog Frequency	S5
S7 Weighted Proportion of Demand Backlogged	S6
S8 Replenishment Frequency	S7
S15 Sources of Total Cost (% Excess Over DP)	S8
S16 Sources of Aggregate Period-End Inventory (% Excess Over DP)	S9
S17 Sources of Aggregate Backlog Cost (% Excess Over DP)	S10
S18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	S11
S19 Backlog Frequency (% Excess Over DP)	S12
S20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	S13
S21 Replenishment Frequency (% Excess Over DP)	S14

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.



Table S3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III.

PULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		466.2	PERCENT OF TOTAL COST= 61.3		MEAN	
LEADTIME	C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	8	16
		4 99	32 64	2 4		
		28.6 71.4	46.8 53.2	45.1 54.9	43.4	56.6
MEAN						
8		12.3 31.1	20.4 23.0	20.3 23.1		
16		16.3 40.3	26.4 30.3	24.7 31.9		
LEADTIME						
2		13.0 32.1	20.1 25.0			
4		15.6 39.4	26.7 28.3			
C (FIX)/C (IN)						
32		13.0 33.8				
64		15.6 37.6				
LEADTIME C (FIX)/C (IN)						
2	32	5.6 14.5				
2	64	7.4 17.6				
4	32	7.4 19.3				
4	64	8.2 20.1				



Table S4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		94.9	PERCENT OF TOTAL COST= 12.5		MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	
MEAN	8	4 99	32 64	2 4	8 16
	16	58.5 41.5	46.3 53.7	47.7 52.3	39.6 60.4
LEADTIME	2	24.0 15.6	19.3 20.3	17.9 21.8	
	4	34.5 25.9	27.0 33.4	29.9 30.5	
C (FIX)/C (IN)	32	27.2 20.5	23.7 24.0		
	64	31.3 21.0	22.5 29.7		
LEADTIME C (FIX)/C (IN)	2	27.4 18.9			
	4	31.1 22.7			
	32	13.3 10.4			
	64	13.9 10.2			
	32	14.1 8.5			
	64	17.2 12.5			

Table S5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		199.0	PERCENT OF TOTAL COST= 26.2		MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	
MEAN	4	99	32	64	8
	50.8	49.2	42.6	57.4	16
LEADTIME	2	20.5	15.8	24.0	39.8
	4	30.3	26.7	33.4	60.2
C (FIX)/C (IN)	2	26.1	21.0	28.6	
	4	24.7	19.5	28.8	
LEADTIME C (FIX)/C (IN)	32	21.3	21.3		
	64	29.5	28.0		
2	32	11.3	11.7		
	64	14.8	13.8		
4	32	10.0	9.6		
	64	14.7	14.1		

Table S6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.0929

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)			LEADTIME			MEAN	
	4	99	32	64		2	4		8	16
	0.1782	0.0076	0.0914	0.0944		0.0946	0.0912		0.0892	0.0966
	0.1718	0.0066	0.0887	0.0897		0.0880	0.0904			
	0.1846	0.0085	0.0941	0.0991		0.1011	0.0920			
	0.1811	0.0080	0.0960	0.0931						
	0.1753	0.0071	0.0868	0.0956						

C (FIX)/C (IN)

0.1756 0.0071  
0.1808 0.0080

LEADTIME C (FIX)/C (IN)

2 32  
2 64  
4 32  
4 64  
0.1838 0.0082  
0.1785 0.0078  
0.1675 0.0060  
0.1830 0.0082

Table S7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0096

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN
4	59	32 64	2 4	8 16
0.1486	0.0041	0.0089 0.0103	0.0092 0.0100	0.0114 0.0087
8	0.1781 0.0047	0.0111 0.0117	0.0103 0.0125	
16	0.1279 0.0039	0.0078 0.0096	0.0086 0.0088	
LEADTIME				
2	0.1345 0.0041	0.0091 0.0092		
4	0.1547 0.0042	0.0087 0.0114		
C (FIX)/C (IN)				
32	0.1355 0.0038			
64	0.1537 0.0045			
LEADTIME C (FIX)/C (IN)				
2	0.1319 0.0042			
4	0.1371 0.0041			
8	0.1392 0.0034			
16	0.1702 0.0050			



Table S8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2770

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	8
0.2801	0.2739		0.3308	0.2232	0.2901	0.2639	0.2163
0.2231	0.2096		0.2460	0.1867	0.2156	0.2171	
0.3371	0.3382		0.4155	0.2598	0.3645	0.3108	
0.2909	0.2893		0.3578	0.2224			
0.2693	0.2586		0.3037	0.2241			
0.3311	0.3304						
0.2291	0.2174						
0.3522	0.3633						
0.2295	0.2152						
0.3100	0.2974						
0.2286	0.2197						

C (FIX)/C (IN)

LEADTIME C (FIX)/C (IN)

Table S15  
 SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL III  
 FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

OVERALL AGGREGATE FOR SYSTEM=			SOURCES OF TOTAL COST (% EXCESS OVER DP)			MEAN
5.8						
MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME	
	4	99	32	64		
8	4.3	6.5	5.1	6.2	6.0	5.4
16	4.1	7.2	6.7	5.4	5.5	6.4
LEADTIME						
	2	4				
2	4.1	6.7	5.9	5.5	5.7	6.0
4	4.2	7.1	6.1	5.9		
C (PIX)/C (IN)						
32	4.3	7.1				
64	4.1	6.8				
LEADTIME C (PIX)/C (IN)						
2	4.7	6.6				
2	3.6	6.8				
4	3.9	7.4				
4	4.5	6.7				



Table S17

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE BACKLOG COST  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		-6.5											
		C(OUT)/C(IN)		C(PIX)/C(IN)		LEADTIME							
		4	99	32	64	2	4	8	16				
		-3.2	-10.9	-12.0	-1.2	-1.1	-11.0	-14.7	-0.2				
		-6.9	-24.5	-15.2	-14.3	-15.0	-14.5						
		-0.3	-0.0	-9.6	8.9	9.7	-8.3						
		0.2	-2.7	0.2	-2.3								
		-5.9	-17.7	-22.1	-0.3								
		-6.1	-19.4										
		-0.4	-2.3										
		2.3	-2.3										
		-1.8	-3.1										
		-12.9	-33.7										
		0.7	-1.6										

MEAN

8

16

LEADTIME

2

4

C(PIX)/C(IN)

32

64

LEADTIME C(PIX)/C(IN)

2

2

4

4





Table S19  
 SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL III  
 FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

OVERALL AGGREGATE FOR SYSTEM=		-7.3		C (OUT)/C (IN)		C (PIX)/C (IN)		LEADTIME		MEAN	
		4	99			32	64	2	4	8	16
				-6.8	-17.7		-8.5	-6.2	-5.0	-9.6	-10.2
											-4.5
							</				

Table S20

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=			-6.5						MEAN		
C (OUT)/C (IN)			C (FIX)/C (IN)			LEADTIME					
4	99		32	64		2	4		8	16	
-3.2 -10.9			-12.0 -1.2			-1.1 -11.0			-14.7 -0.2		
MEAN											
8			-6.9 -24.5			-15.2 -14.3			-15.0 -14.5		
16			-0.3 -0.0			-9.6 8.9			9.7 -8.3		
LEADTIME											
2			0.2 -2.7			0.2 -2.3					
4			-5.9 -17.7			-22.1 -0.3					
C (FIX)/C (IN)											
32			-6.1 -19.4								
64			-0.4 -2.3								
LEADTIME C (FIX)/C (IN)											
2	32		2.3 -2.3								
2	64		-1.8 -3.1								
4	32		-12.9 -33.7								
4	64		0.7 -1.6								

Table S21

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL II:

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

OVERALL AGGREGATE FOR SYSTEM=			7.1			REPLENISHMENT FREQUENCY (% EXCESS OVER DP)		
MEAN	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	MEAN	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	MEAN
8	9.5	-2.0	0.6	7.9	32	64	2	4
16	15.1	4.3	15.7	0.6	32	64	2	4
2	12.2	2.9	13.5	-1.2	32	64	2	4
4	13.4	0.5	5.4	8.6	32	64	2	4
C (PIX)/C (IN)	15.3	4.4	9.4	-2.0	32	64	2	4
32	17.0	10.3	5.7	-7.5	32	64	2	4
64	13.5	-2.0	13.4	4.1	32	64	2	4
LEADTIME C (PIX)/C (IN)	2	32	2	64	32	64	2	4



## Appendix T

### Multi-item Data for the (24,24) Statistical Power Approximation

#### Model III

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled with Statistical Information from  
a 24-Period Demand History, with Revision Every 24 Periods,  
Using Regression Estimates of Demand Means and Variances.

	<u>page</u>
Table T1 Sources of Total Cost	T1
T3 Sources of Aggregate Period-End Inventory	T2
T4 Sources of Aggregate Backlog Cost	T3
T5 Sources of Aggregate Replenishment Cost	T4
T6 Backlog Frequency	T5
T7 Weighted Proportion of Demand Backlogged	T6
T8 Replenishment Frequency	T7
T15 Sources of Total Cost (% Excess Over DP)	T8
T16 Sources of Aggregate Period-End Inventory (% Excess Over DP)	T9
T17 Sources of Aggregate Backlog Cost (% Excess Over DP)	T10
T18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	T11
T19 Backlog Frequency (% Excess Over DP)	T12
T20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	T13
T21 Replenishment Frequency (% Excess Over DP)	T14

Note: For corresponding data in MacCormick (1974), see the table  
of the same number in his Appendices K and L.

Table T1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 824.4				SOURCES OF TOTAL COST			
MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN		C (OUT)/C (IN)	C (FIX)/C (IN)
				2	4		
8	15.4	27.0	19.4	22.9	19.8	22.6	
16	21.3	36.3	26.4	31.2	26.7	30.9	
LEADTIME							
2	17.4	29.1	21.2	25.3			
4	19.3	34.2	24.6	28.9			
C (FIX)/C (IN)							
32	16.4	29.5					
64	20.3	33.8					
LEADTIME C (FIX)/C (IN)							
2	7.7	13.5					
4	9.7	15.6					
8	8.7	16.0					
16	10.7	18.2					

42.4 57.6

Table T3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE PERIOD-END INVENTORY						
OVERALL AGGREGATE FOR SYSTEM=		496.6	PERCENT OF TOTAL COST=		60.2	
	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	MEAN		
	4	99	32	64	2	4
					8	16
	28.7	71.3	46.4	53.6	45.0	55.0
					44.1	55.9
	12.5	31.6	20.8	23.2	20.5	23.6
	16.2	39.7	25.6	30.3	24.5	31.4
	13.0	32.0	20.2	24.9		
	15.7	39.3	26.2	28.7		
	12.9	33.5				
	15.8	37.8				
	5.6	14.6				
	7.4	17.5				
	7.3	18.9				
	8.4	20.4				

Table T4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, RE-USED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		128.5	PERCENT OF TOTAL COST=		15.6	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	
4	99	45.6	32	64	2	4
			48.4	51.6	44.7	55.3
MEAN						
8	16	18.9	21.1	19.9	20.1	17.0
			26.7	33.3	28.5	31.4
LEADTIME						
2	4	20.7	24.0	22.7	22.0	23.0
			24.9	30.4	25.8	29.5
C (FIX)/C (IN)						
32	64	21.9	26.6			
			23.7	27.8		
LEADTIME C (FIX)/C (IN)						
2	32	10.3	12.4			
			10.5	11.6		
4	64	11.6	14.1			
			13.3	16.2		
LEADTIME C (FIX)/C (IN)						
8	16	40.0	60.0			
			40.0	60.0		



AD-A050 140

NORTH CAROLINA UNIV AT CHAPEL HILL GRADUATE SCHOOL OF--ETC F/G 15/5  
(S,S) INVENTORY POLICIES IN A NONSTATIONARY DEMAND ENVIRONMENT.--ETC(U)  
APR 77 R L KAUFMAN.

DAAG29-76-G-0323

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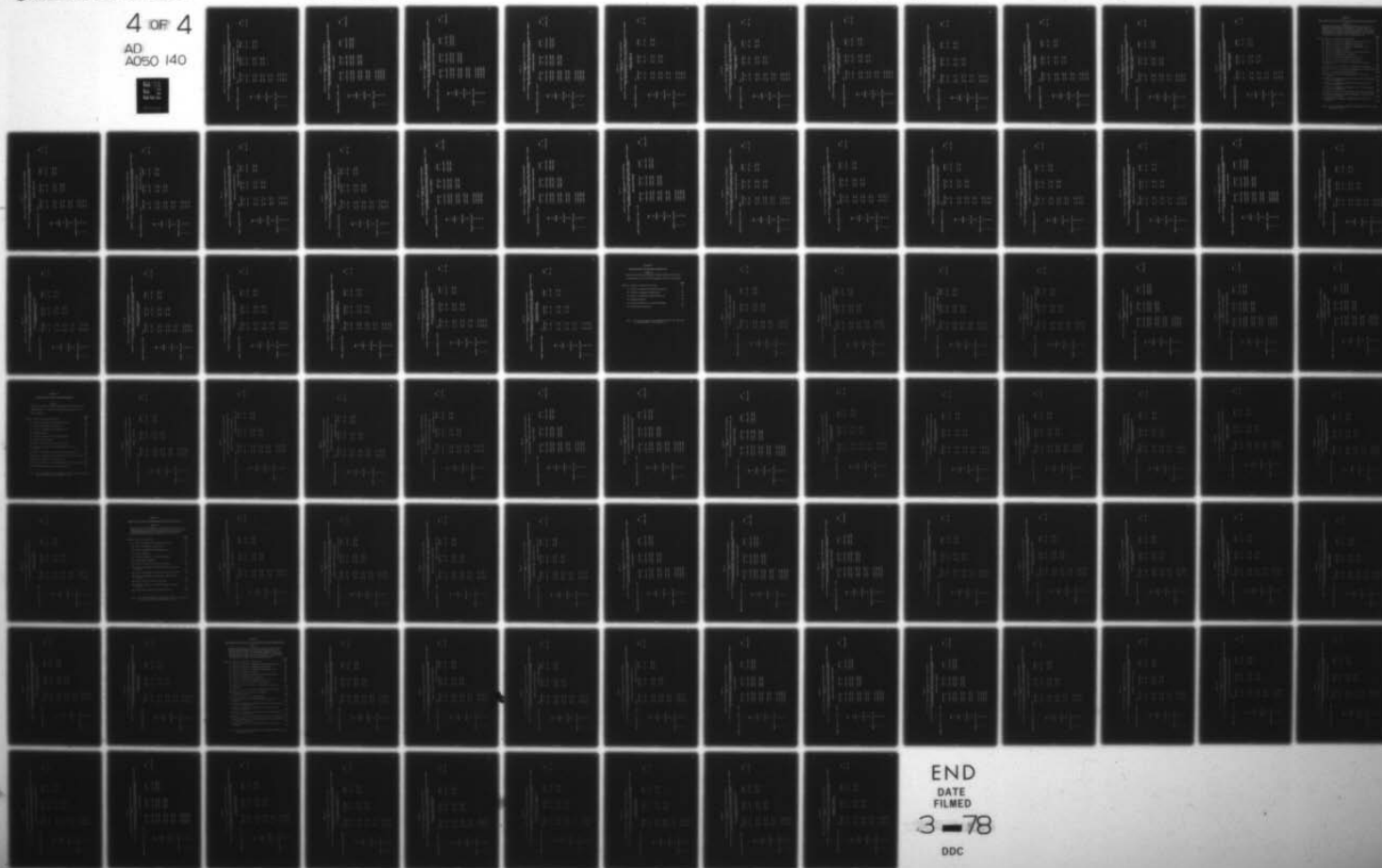






Table T6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 0.0952				BACKLOG FREQUENCY			
C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
4	99	32	64	2	4	8	16
0.1789 0.0115		0.0952 0.0952		0.0952 0.0952		0.0920 0.0984	
0.1736 0.0104		0.0912 0.0929		0.0890 0.0951			
0.1841 0.0127		0.0991 0.0976		0.1014 0.0954			
0.1792 0.0112		0.0974 0.0929					
0.1785 0.0119		0.0929 0.0975					
0.1788 0.0115							
0.1789 0.0116							
0.1832 0.0116							
0.1752 0.0107							
0.1745 0.0113							
0.1826 0.0125							



Table T7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0130

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN	
				2	8
4	99	32	64	4	16
0.1526	0.0074	0.0126	0.0134	0.0116	0.0144
0.1899	0.0086	0.0155	0.0157	0.0133	0.0179
0.1340	0.0067	0.0111	0.0123	0.0108	0.0126
0.1387	0.0065	0.0118	0.0115		
0.1666	0.0082	0.0134	0.0153		
0.1464	0.0072				
0.1588	0.0075				
0.1374	0.0067				
0.1399	0.0063				
0.1554	0.0076				
0.1778	0.0088				

C (FIX)/C (IN)

LEADTIME C (FIX)/C (IN)

Table T8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL III  
 STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

REPLENISHMENT FREQUENCY				
OVERALL AGGREGATE FOR SYSTEM= 0.2778				
MEAN	C (OUT) /C (IN)	C (FIX) /C (IN)	LEADTIME	MEAN
4	99	32 64	2 4	8 16
0.2810	0.2747	0.3327 0.2229	0.2880 0.2676	0.2155 0.3401
8	0.2225 0.2086	0.2442 0.1869	0.2149 0.2161	
16	0.3395 0.3407	0.4213 0.2589	0.3611 0.3191	
LEADTIME				
2	0.2893 0.2867	0.3540 0.2220		
4	0.2727 0.2626	0.3115 0.2238		
C (FIX) /C (IN)				
32	0.3327 0.3328			
64	0.2293 0.2165			
LEADTIME C (FIX) /C (IN)				
2	0.3489 0.3592			
2	0.2298 0.2143			
4	0.3165 0.3065			
4	0.2289 0.2187			









Table T18

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		6.2																	
		C (OUT)/C (IN)		C (PIX)/C (IN)		LEADTIME		LEADTIME		MEAN									
		4	99	32	64	2	4	2	4	8	16								
		12.1	0.7	10.2	3.4	4.3	8.3	4.6	7.3										
		11.0	-1.4	-0.2	8.0	-0.6	10.4												
		12.8	2.2	17.3	0.3	7.6	6.9												
		9.9	-0.9	12.3	-1.3														
		14.4	2.6	8.0	8.5														
		15.8	5.2																
		9.5	-2.4																
		15.8	9.0																
		5.8	-7.9																
		15.8	1.0																
		13.5	3.7																

LEADTIME C (PIX)/C (IN)

2 32  
2 64  
4 32  
4 64

C (PIX)/C (IN)

32  
64

LEADTIME

2 4

MEAN

8 16

Table T19

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			BACKLOG FREQUENCY (% EXCESS OVER DP)			MEAN		
-5.0			C (OUT)/C (IN)			C (FIX)/C (IN)		
MEAN	LEADTIME	C (FIX)/C (IN)	4	99	32	64	2	4
			8	16	-7.4	-2.7		
8			-8.4	14.0	-7.9	-6.9	-9.5	-5.3
16			-4.6	36.5	-1.6	-3.8	0.6	-6.0
LEADTIME			-5.7	22.8	-1.7	-7.1		
			-7.3	27.9	-7.8	-3.6		
C (FIX)/C (IN)			-6.1	23.9				
			-6.9	26.9				
LEADTIME	C (FIX)/C (IN)		-3.1	26.6				
			-8.3	18.9				
			-9.2	21.1				
			-5.4	34.7				





Table T21

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL III  
 STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			REPLENISHMENT FREQUENCY (% EXCESS OVER DP)			MEAN
7.4			C (OUT)/C (IN)			
LEADTIME	C (FIX)/C (IN)	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME
			4	99	32 64	
2	11.6	2.0	13.2	2.0	10.2 3.4	6.6 8.2
4	14.8	2.1	15.9	5.0	17.3 0.3	12.3 7.9
32	15.8	5.2	9.2	-2.5	-0.2 8.0	-1.8 8.7
64	9.5	-2.4	15.9	5.0	17.3 0.3	12.3 7.9
2	15.8	9.0	13.2	2.0	10.2 3.4	6.6 8.2
2	5.8	-7.9	15.9	5.0	17.3 0.3	-1.8 8.7
4	15.8	1.0	9.2	-2.5	-0.2 8.0	-1.8 8.7
4	13.5	3.7	15.9	5.0	17.3 0.3	12.3 7.9

## Appendix U

### Multi-item Forecasts for the (24,24) Statistical Power Approximation

#### Model III

Forecasting Properties of Inventory System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information, Revision Taking Place Every 24 Periods Using a 24-Period Demand History and Regression Estimates of Demand Means and Variances. Forecasts Made at Each Revision Using a 24-Period Demand History.

	<u>page</u>
Table U1 Sources of Forecast of Total Cost	U1
U2 Sources of Forecast of Aggregate Period-End Inventory	U2
U3 Sources of Forecast of Aggregate Backlog Cost	U3
U4 Sources of Forecast of Aggregate Replenishment Cost	U4
U5 Forecast of Backlog Frequency	U5
U6 Forecast of Weighted Proportion of Demand Backlogged	U6
U7 Forecast of Replenishment Frequency	U7
U8 Sources of Variance of Forecast of Total Cost	U8
U9 Sources of S.D. of Forecast of Total Cost	U9
U10 Sources of S.D. of Forecast of Aggregate Period-End Inventory	U10
U11 Sources of S.D. of Forecast of Aggregate Backlog Cost	U11
U12 Sources of S.D. of Forecast of Aggregate Replenishment Cost	U12
U13 S.D. of Forecast of Backlog Frequency	U13
U14 Sources of Total Cost (% Underestimate of Actual by Forecast)	U14
U15 Sources of Aggregate Period-End Inventory (% Underestimate of Actual by Forecast)	U15
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Note: The corresponding appendices in MacCormick (1974), are his Appendices T and U.

Table U1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL III  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF TOTAL COST			
OVERALL AGGREGATE FOR SYSTEM= 735.4			
	C(OUT)/C(IN)	C(FIX)/C(IN)	MEAN
	4 99	32 64	8 16
	38.4 61.6	45.5 54.5	47.7 52.3
			42.3 57.7
MEAN			
8	16.1 26.3	19.2 23.1	20.3 22.0
16	22.4 35.3	26.3 31.4	27.4 30.3
LEADTIME			
2	18.6 29.1	21.5 26.2	
4	19.8 32.5	24.0 28.3	
C(FIX)/C(IN)			
32	17.0 28.5		
64	21.5 33.0		
LEADTIME C(FIX)/C(IN)			
2	8.2 13.4		
2	10.5 15.7		
4	8.8 15.2		
4	11.0 17.3		

Table U2

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS MODEL III						
FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES						
SOURCES OF AGGREGATE PERIOD-END INVENTORY						
OVERALL AGGREGATE FOR SYSTEM=		475.0	PERCENT OF TOTAL COST=		64.6	
		C (OUT)/C (IN)		C (FIX)/C (IN)		MEAN
		4	99	32	64	
		28.0	72.0	46.5	53.5	43.8 56.2
MEAN	8	12.1	31.6	20.7	23.1	20.9 22.9
	16	15.9	40.3	25.8	30.5	25.2 31.0
LEADTIME	2	13.2	32.9	20.7	25.4	
	4	14.9	39.0	25.8	28.1	
C (FIX)/C (IN)		12.6	33.9			
		15.5	38.0			
LEADTIME	2	5.7	15.0			
	4	7.5	17.9			
C (FIX)/C (IN)	32	6.9	18.9			
	64	8.0	20.1			







Table U5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL III  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 0.0875					BACKLOG FREQUENCY				
MEAN	LEADTIME	C (OUT)/C (IN)		C (FIX)/C (IN)	LEADTIME		MEAN		
		4	99		2	4		8	16
		0.1714	0.0035	0.0860	0.0890	0.0892	0.0857	0.0846	0.0904
8		0.1659	0.0032	0.0832	0.0859	0.0821	0.0870		
16		0.1770	0.0037	0.0887	0.0920	0.0963	0.0844		
		0.1744	0.0040	0.0897	0.0887				
		0.1685	0.0030	0.0822	0.0892				
		0.1687	0.0032						
		0.1742	0.0037						
		0.1751	0.0043						
		0.1736	0.0037						
		0.1623	0.0022						
		0.1747	0.0037						

Table U6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0062

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME			MEAN
	4	99	32	64	2	4	8	16
0.1257	0.0014	0.0059	0.0066	0.0061	0.0063	0.0067	0.0082	0.0056
0.1551	0.0015	0.0070	0.0079	0.0061	0.0061	0.0067	0.0082	0.0056
0.1110	0.0014	0.0053	0.0059	0.0056	0.0070	0.0058	0.0054	0.0056
0.1197	0.0015	0.0061	0.0061	0.0061	0.0061	0.0061	0.0061	0.0061
0.1317	0.0013	0.0056	0.0070	0.0056	0.0070	0.0056	0.0070	0.0056
0.1181	0.0013	0.1181	0.0013	0.1181	0.0013	0.1181	0.0013	0.1181
0.1332	0.0015	0.1332	0.0015	0.1332	0.0015	0.1332	0.0015	0.1332
0.1145	0.0017	0.1145	0.0017	0.1145	0.0017	0.1145	0.0017	0.1145
0.1248	0.0013	0.1248	0.0013	0.1248	0.0013	0.1248	0.0013	0.1248
0.1217	0.0009	0.1217	0.0009	0.1217	0.0009	0.1217	0.0009	0.1217
0.1417	0.0016	0.1417	0.0016	0.1417	0.0016	0.1417	0.0016	0.1417



Table U7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL III  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2769

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME			MEAN
	4	32	64	4	2	4	8	16
	0.2808	0.2730	0.3310	0.2228	0.2861	0.2678		0.2145 0.3393
8	0.2220	0.2071	0.2434	0.1856	0.2119	0.2172		
16	0.3397	0.3389	0.4186	0.2599	0.3603	0.3183		
LEADTIME								
2	0.2884	0.2837	0.3515	0.2206				
4	0.2733	0.2622	0.3106	0.2249				
C (FIX)/C (IN)								
32	0.3322	0.3299						
64	0.2295	0.2161						
LEADTIME C (FIX)/C (IN)								
2	0.3482	0.3548						
2	0.2285	0.2127						
4	0.3161	0.3050						
4	0.2304	0.2195						

Table U8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF VARIANCE OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM= 1489.1

C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN
4 99	32 64	2 4	8 16
13.2 86.8	45.7 54.3	37.0 63.0	38.1 61.9

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64

2.0 17.3

3.1 14.6

4.2 22.2

4.0 32.6

19.1 19.0

26.6 35.2

19.3 17.7

26.4 36.6

6.2 39.6

7.1 47.2

13.6 24.5

23.4 38.5











Table U13

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

S.D. OF BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1868

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN	
	4	99	32	64	2	4	8	16
	0.1832	0.0368	0.1326	0.1317	0.1199	0.1433	0.1392	0.1247
8	0.1368	0.0255	0.0968	0.1000	0.0828	0.1119		
16	0.1218	0.0265	0.0906	0.0856	0.0867	0.0895		
LEADTIME								
2	0.1171	0.0259	0.0862	0.0834				
4	0.1409	0.0261	0.1007	0.1019				
C (FIX) / C (IN)								
32	0.1303	0.0244						
64	0.1288	0.0275						
LEADTIME C (FIX) / C (IN)								
2	0.0842	0.0183						
2	0.0813	0.0183						
4	0.0994	0.0161						
4	0.0998	0.0206						







Table U16

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL III

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST  
(% UNDERESTIMATE OF ACTUAL)

OVERALL AGGREGATE FOR SYSTEM=										52.1
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN		
		4	99	32	64	2	4	8	16	
		17.7	81.0	53.4	50.9	47.2	56.0	52.2	52.0	
		18.3	82.6	54.7	49.8	49.3	54.4			
		17.2	79.9	52.5	51.6	46.0	57.1			
		13.7	76.2	48.1	46.4					
		20.9	84.7	58.1	54.2					
		19.3	81.4							
		16.1	80.5							
		16.7	74.0							
		10.8	78.5							
		21.7	88.0							
		20.3	81.9							

		C (FIX)/C (IN)		LEADTIME	
		32	64	2	4
		19.3	81.4	13.7	76.2
		16.1	80.5	20.9	84.7
		16.7	74.0	19.3	81.4
		10.8	78.5	16.1	80.5
		21.7	88.0	16.7	74.0
		20.3	81.9	10.8	78.5

LEADTIME C (FIX)/C (IN)

2 32  
2 64  
4 32  
4 64

C (FIX)/C (IN)

32  
64

LEADTIME

2  
4

MEAN

8  
16



Table U18

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IIX

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY: CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

BACKLOG FREQUENCY  
(% UNDERESTIMATE OF ACTUAL)

OVERALL AGGREGATE FOR SYSTEM=		8.1																	
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		LEADTIME		C (FIX)/C (IN)		LEADTIME		C (FIX)/C (IN)		LEADTIME		C (FIX)/C (IN)	
		4 99		32 64		2 4		2 4		32 64		2 4		32 64		2 4		32 64	
		4.2 69.8		9.7 6.6		6.3 10.0		6.3 10.0		9.7 6.6		6.3 10.0		9.7 6.6		6.3 10.0		9.7 6.6	
		4.5 69.0		8.8 7.5		7.7 8.4		7.7 8.4		8.8 7.5		7.7 8.4		8.8 7.5		7.7 8.4		8.8 7.5	
		3.9 70.4		10.5 5.8		5.0 11.5		5.0 11.5		10.5 5.8		5.0 11.5		10.5 5.8		5.0 11.5		10.5 5.8	
		2.7 64.1		7.9 4.6						7.9 4.6				7.9 4.6				7.9 4.6	
		5.6 75.0		11.5 8.5						11.5 8.5				11.5 8.5				11.5 8.5	
		5.7 71.9																	
		2.6 67.7																	
		4.4 63.2																	
		0.9 65.1																	
		7.0 80.7																	
		4.3 69.9																	

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

4

8

16

32

64

128



Table U19

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL II:  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=		52.1		WEIGHTED PROPORTION OF DEMAND BACKLOGGED (% UNDERESTIMATE OF ACTUAL)		MEAN	
LEADTIME	C (PIX)/C (IN)	C (OUT)/C (IN)	C (PIX)/C (IN)	LEADTIME	C (PIX)/C (IN)	LEADTIME	MEAN
2	32	17.7	81.0	47.2	56.0	52.2	52.0
4	64	18.3	82.6	49.3	54.4		
8	16	17.2	79.9	46.0	57.1		
2	32	13.7	76.2	48.1	46.4		
4	64	20.9	84.7	58.1	54.2		
2	32	19.3	81.4				
4	64	16.1	80.5				
2	32	16.7	74.0				
4	64	10.8	78.5				
2	32	21.7	88.0				
4	64	20.3	81.9				



## Appendix V

### Multi-item Data for the Power Approximation

#### Model IV

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Optimally with Full Information.

	<u>page</u>
Table V1 Sources of Expected Total Cost	V1
V3 Sources of Aggregate Period-End Inventory	V2
V4 Sources of Aggregate Backlog Cost	V3
V5 Sources of Aggregate Replenishment Cost	V4
V6 Backlog Frequency	V5
V7 Weighted Proportion of Demand Backlogged	V6
V8 Replenishment Frequency	V7

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.

Table VI

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM=	739.2
-------------------------------	-------

LEADLINE	MEAN		
2 4	8	16	
7.2 52.8	42.0	58.0	

MRAN

8

16

LEADTIME

2

47

$$C(FIX)/C(IN)$$

32

73

LEADTIME C(FIX)/C(IN)

2

2

29

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17-1 28-2

21.6 33.1

21.2 26.0

24-1 28-7

19-1 22-9

26-2 31-8

19-8 22-2

27-4 30-6



Table V3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

PULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		433.6	PERCENT OF TOTAL COST=		58.7	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	4	99	32	64	2	4
						8
						16
LEADTIME	30.2	69.8	47.0	53.0	45.9	54.1
						42.1
						57.9
C (FIX)/C (IN)	12.6	29.5	19.8	22.3	19.3	22.8
	17.6	40.3	27.2	30.7	26.6	31.3
LEADTIME	13.9	32.0	21.4	24.5		
	16.2	37.9	25.6	28.5		
C (FIX)/C (IN)	13.7	33.3				
	16.5	36.5				
LEADTIME	6.2	15.1				
	7.7	16.8				
	7.4	18.2				
	8.8	19.7				

Table V4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

## FULL INFORMATION, OPTIMAL CONTROL (DP)

## SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		104.1	PERCENT OF TOTAL COST= 14.1		MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	
MEAN	8	4 99	32 64	2 4	8 16
	16	56.8 43.2	48.7 51.3	45.4 54.6	43.1 56.9
LEADTIME	2	24.1 19.0	21.2 21.9	19.7 23.3	
	4	32.7 24.2	27.5 29.4	25.7 31.3	
C (FIX)/C (IN)	32	25.6 19.7	22.2 23.2		
	64	31.1 23.5	26.5 28.1		
LEADTIME C (FIX)/C (IN)	2	27.2 21.5			
	4	29.6 21.8			
C (FIX)/C (IN)	32	12.3 9.9			
	64	13.3 9.9			
LEADTIME	2	14.9 11.6			
	4	16.2 11.9			

Table V5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, OPTIMAL CONTROL (DP)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		201.5	PERCENT OF TOTAL COST=		27.3	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	8	4 99	32 64	2 4		8 16
	16					
LEADTIME	2	47.8 52.2	39.9 60.1	51.1 48.9		41.2 58.8
	4					
C (FIX)/C (IN)	32	19.7 21.5	15.6 24.6	21.0 20.2		
	64	28.1 30.7	23.4 35.4	30.1 28.7		
LEADTIME	2	24.6 26.5	20.4 30.7			
	4	23.1 25.8	19.6 29.3			
C (FIX)/C (IN)	32	19.3 20.6				
	64	28.4 31.6				
LEADTIME	2	9.9 10.4				
	4	14.7 16.0				
C (FIX)/C (IN)	32	9.4 10.2				
	64	13.7 15.6				

Table V6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

## FULL INFORMATION, OPTIMAL CONTROL (DP)

## BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1003

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN	
				8	16
4	99	32 64	2 4		
0.1913	0.0092	0.1001 0.1004	0.0996 0.1009	0.0997	0.1008
0.1903	0.0091	0.1000 0.0994	0.0994 0.1000		
0.1924	0.0093	0.1002 0.1014	0.0998 0.1018		
LEADTIME					
2	0.1901 0.0091	0.0998 0.0995			
4	0.1926 0.0092	0.1004 0.1013			
C (FIX)/C (IN)					
32	0.1911 0.0092				
64	0.1916 0.0092				
LEADTIME C (FIX)/C (IN)					
2	0.1903 0.0092				
2	0.1899 0.0091				
4	0.1918 0.0091				
4	0.1934 0.0093				



Table V7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, OPTIMAL CONTROL (DP)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0105

MEAN  
8 16  
0.0136 0.0090

C (OUT)/C (IN) C (FIX)/C (IN) LEADTIME  
4 99 32 64 2 4  
0.1540 0.0047 0.0102 0.0108 0.0096 0.0115

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

2

4

4

0.1334 0.0043

0.1448 0.0043

0.1617 0.0051

0.1761 0.0052

0.1475 0.0047

0.1605 0.0048

0.1391 0.0043 0.0093 0.0098

0.1689 0.0051 0.0112 0.0118

0.1959 0.0062 0.0134 0.0138

0.1331 0.0040 0.0087 0.0093

0.0125 0.0147

0.0081 0.0099

Table V8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, OPTIMAL CONTROL (DP)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2754

MEAN	C (OUT) / C (IN)	C (FIX) / C (IN)	LPADTIME	MEAN	
				8	16
4	99	32 64	2 4		
0.2640	0.2867	0.3143 0.2364	0.2813 0.2694	0.2273	0.3234
0.2166	0.2381	0.2607 0.1940	0.2314 0.2233		
0.3115	0.3353	0.3680 0.2788	0.3312 0.3155		
0.2723	0.2903	0.3207 0.2419			
0.2558	0.2831	0.3079 0.2309			
0.3042	0.3245				
0.2239	0.2489				
0.3131	0.3284				
0.2315	0.2523				
0.2953	0.3206				
0.2164	0.2455				

C (FIX) / C (IN)

LEADTIME C (FIX) / C (IN)

## Appendix W

### Multi-item Data for the Power Approximation

#### Model IV

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled Approximately Optimally with  
Full Information.

	<u>page</u>
Table W1 Sources of Expected Total Cost	W1
W3 Sources of Aggregate Period-End Inventory	W2
W4 Sources of Aggregate Backlog Cost	W3
W5 Sources of Aggregate Replenishment Cost	W4
W6 Backlog Frequency	W5
W7 Weighted Proportion of Demand Backlogged	W6
W8 Replenishment Frequency	W7
W15 Sources of Total Cost (% Excess Over DP)	W8
W16 Sources of Aggregate Period-End Inventory (% Excess Over DP)	W9
W17 Sources of Aggregate Backlog Cost (% Excess Over DP)	W10
W18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	W11
W19 Backlog Frequency (% Excess Over DP)	W12
W20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	W13
W21 Replenishment Frequency (% Excess Over DP)	W14

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Table W1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM=		758.7																	
		C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN											
		4	99	32	64	2	4	8	16										
		38.4	61.6	45.1	54.9	47.1	52.9	42.1	57.9										
		16.1	26.1	19.1	23.1	19.9	22.3												
		22.4	35.5	26.1	31.8	27.3	30.6												
		18.3	28.8	21.1	26.1														
		20.1	32.7	24.0	28.8														
		16.9	28.2																
		21.5	33.3																
		8.0	13.1																
		10.3	15.7																
		8.9	15.1																
		11.2	17.6																

LEADTIME C (FIX) / C (IN)

2	32	8.0	13.1
2	64	10.3	15.7
4	32	8.9	15.1
4	64	11.2	17.6



Table W3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

PULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		460.7		PERCENT OF TOTAL COST= 60.7		MEAN
C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		
4	99	32	64	2	4	8 16
28.8	71.2	47.2	52.8	46.1	53.9	42.2 57.8

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

2

4

4

6.1 15.4

7.3 17.3

7.2 18.5

8.2 20.0

13.3 33.9

15.6 37.3

13.4 32.7

15.4 38.5

11.9 30.3

16.9 40.9

20.0 22.2

27.1 30.7

21.5 24.6

25.6 28.3

19.5 22.7

26.6 31.2

Table W4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

## FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

## SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		96.8	PERCENT OF TOTAL COST= 12.8		MEAN	
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	8	16
		4	32	64	2	4
		58.7	41.3	47.5	52.5	45.4
					54.6	42.8
						57.2
MEAN						
8		25.4	17.4	19.4	23.5	19.6
16		33.3	23.9	28.1	29.1	25.9
LEADTIME						31.3
2		26.8	18.6	21.6	23.9	
4		31.9	22.7	25.9	28.7	
C (FIX)/C (IN)						
32		27.6	19.6			
64		30.8	21.7			
LEADTIME C (FIX)/C (IN)						
2		12.6	9.0			
4		14.2	9.6			
8		15.2	10.7			
16		16.6	12.0			

Table W5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM=		201.1	PERCENT OF TOTAL COST= 26.5		MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	
MEAN	4	99	32	64	8
	16				16
LEADTIME	2	50.8	49.2	39.4	60.6
	4			50.3	49.7
C (FIX)/C (IN)	32	21.1	20.5	16.7	24.9
	64	29.6	28.7	22.7	35.7
LEADTIME	2	25.5	24.8	19.8	30.5
	4	25.3	24.4	19.5	30.2
C (FIX)/C (IN)	32	20.0	19.3		
	64	30.7	29.9		
LEADTIME	2	10.1	9.7		
	4	15.4	15.1		
C (FIX)/C (IN)	32	9.9	9.6		
	64	15.3	14.8		

41.6 58.4

Table W6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

PULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.0946

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME			MEAN
		4	99	32	64	2	4	8
8	0.1814	0.0078	0.0933	0.0959	0.0943	0.0949	0.0943	0.0589
16	0.1823	0.0075	0.0914	0.0984	0.0944	0.0953	0.0946	
	0.1806	0.0081	0.0952	0.0935	0.0941	0.0946		
	0.1809	0.0077	0.0929	0.0956				
	0.1819	0.0079	0.0936	0.0962				

C (FIX)/C (IN)

32 0.1790 0.0075  
64 0.1838 0.0081

LEADTIME C (FIX)/C (IN)

2 0.1784 0.0075  
2 0.1834 0.0078  
4 0.1797 0.0075  
4 0.1842 0.0083



Table W7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0098

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
0.1480	0.0042		0.0093	0.0103	0.0089	0.0107	0.0126
0.1920	0.0053		0.0114	0.0138	0.0115	0.0137	
0.1259	0.0037		0.0083	0.0085	0.0076	0.0092	

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

2

4

4

0.1268 0.0037

0.1433 0.0039

0.1538 0.0043

0.1679 0.0049

0.1403 0.0040

0.1556 0.0044

0.1351 0.0038

0.1608 0.0046

0.0084 0.0093

0.0101 0.0112

Table W8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM = 0.2737

MEAN	C (OUT) / C (IN)	C (FIX) / C (IN)			LEADTIME			MEAN
	4	99	32	64	2	4	8	16
	0.2780	0.2694	0.3092	0.2382	0.2756	0.2718		0.2291 0.3183
8	0.2326	0.2256	0.2624	0.1958	0.2302	0.2280		
16	0.3235	0.3132	0.3560	0.2806	0.3210	0.3156		
LEADTIME	C (FIX) / C (IN)	C (FIX) / C (IN)			LEADTIME			MEAN
	2	4	32	64	2	4	8	16
	0.2795	0.2717	0.3118	0.2395	0.2756	0.2718		0.2291 0.3183
	0.2766	0.2671	0.3067	0.2370	0.2302	0.2280		
	0.3145	0.3039			0.3210	0.3156		
	0.2415	0.2349						
LEADTIME	C (FIX) / C (IN)	C (FIX) / C (IN)			LEADTIME			MEAN
	2	4	32	64	2	4	8	16
	0.3171	0.3064			0.2756	0.2718		0.2291 0.3183
	0.2419	0.2370			0.2302	0.2280		
	0.3120	0.3014			0.3210	0.3156		
	0.2412	0.2328						



Table W16

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE PERIOD-END INVENTORY  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		6.3		C (OUT)/C (IN)		C (PIX)/C (IN)		LEADTIME		MEAN	
	4	99	32	64	2	4	8	16			
	1.5	8.3	6.6	5.9	6.7	5.9	6.5	6.1			
MEAN	8	0.6	9.0	7.6	5.5	7.1	5.9				
	16	2.1	7.9	5.9	6.3	6.5	5.8				
LEADTIME	2	2.2	8.7	7.0	6.5						
	4	0.9	8.0	6.3	5.4						
C (PIX)/C (IN)		3.2	8.1								
	32	0.0	8.6								
	64										
LEADTIME C (PIX)/C (IN)		3.9	8.3								
	2	0.7	9.2								
	4	2.6	7.9								
	4	-0.6	8.1								





Table W18

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FULL INFORMATION, APPROXIMATELY OPTIMAL CONTROL (PA)

SOURCES OF AGGREGATE REPLENISHMENT COST  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=-0.2																			
C (OUT)/C (IN)					C (FIX)/C (IN)					LEADTIME					MEAN				
4 99					32 64					2 4					8 16				
6.1 -5.9					-1.6 0.8					-1.7 1.4					0.8 -0.9				
MEAN																			
8					7.2 -5.0					0.7 0.9					-0.9 2.6				
16					5.3 -6.5					-3.3 0.7					-2.3 0.6				
LEADTIME																			
2					3.2 -6.3					-2.8 -1.0									
4					9.1 -5.5					-0.4 2.6									
C (PIX)/C (IN)																			
32					3.4 -6.3														
64					7.9 -5.6														
LEADTIME C (FIX)/C (IN)																			
2					1.3 -6.7														
2					4.5 -6.1														
4					5.7 -6.0														
4					11.5 -5.2														









## Appendix X

### Multi-item Data for the (24,24) Statistical Power Approximation

#### Model IV

System of 16 Items with Negative Binomial Demand Distributions  
(Variance/Mean = 3) Controlled with Statistical Information from  
a 24-Period Demand History, with Revision Every 24 Periods,  
Using Regression Estimates of Demand Means and Variances.

	<u>page</u>
Table X1 Sources of Total Cost	X1
X3 Sources of Aggregate Period-End Inventory	X2
X4 Sources of Aggregate Backlog Cost	X3
X5 Sources of Aggregate Replenishment Cost	X4
X6 Backlog Frequency	X5
X7 Weighted Proportion of Demand Backlogged	X6
X8 Replenishment Frequency	X7
X15 Sources of Total Cost (% Excess Over DP)	X8
X16 Sources of Aggregate Period-End Inventory (% Excess Over DP)	X9
X17 Sources of Aggregate Backlog Cost (% Excess Over DP)	X10
X18 Sources of Aggregate Replenishment Cost (% Excess Over DP)	X11
X19 Backlog Frequency (% Excess Over DP)	X12
X20 Weighted Proportion of Demand Backlogged (% Excess Over DP)	X13
X21 Replenishment Frequency (% Excess Over DP)	X14

Note: For corresponding data in MacCormick (1974), see the table  
of the same number in his Appendices K and L.

Table XI

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=		817.9		SOURCES OF TOTAL COST		MEAN
		C (OUT)/C (IN)		C (FIX)/C (IN)		
		4	99	32	64	8 16
		37.2	62.8	45.7	54.3	46.5 53.5
						42.4 57.6
MEAN						
8		15.6	26.8	19.4	22.9	19.7 22.7
16		21.6	36.0	26.2	31.4	26.8 30.9
LEADTIME						
2		17.5	28.9	21.0	25.4	
4		19.7	33.8	24.7	28.9	
C (FIX)/C (IN)						
32		16.5	29.2			
64		20.7	33.6			
LEADTIME C (FIX)/C (IN)						
2		7.7	13.3			
2		9.3	15.6			
4		8.8	15.9			
4		10.9	18.0			

Table X3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		491.7	PERCENT OF TOTAL COST=		60.1	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)		LEADTIME	
MEAN	4	99	32	64	2	4
	8					16
LEADTIME	2	29.2	70.8	47.1	52.9	45.6
	4					54.4
C (FIX)/C (IN)	32	12.2	30.5	20.4	22.3	19.5
	64	17.0	40.3	26.8	30.6	26.1
LEADTIME	2	13.3	32.3	21.2	24.4	23.2
	4	15.9	38.5	25.9	28.4	31.2
C (FIX)/C (IN)	32	13.4	33.8			
	64	15.8	37.1			
LEADTIME	2	6.0	15.2			
	4	7.3	17.1			
C (FIX)/C (IN)	32	7.4	18.6			
	64	8.5	20.0			
						42.7
						57.3



Table X4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		124.1	PERCENT OF TOTAL COST=		15.2	MEAN
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
MEAN	4	99	32	64	2	4
	8					16
LEADTIME	46.9	53.1	49.6	50.4	43.1	56.9
	20.3	22.4	20.3	22.4	18.6	24.1
C (FIX)/C (IN)	26.6	30.7	29.3	28.0	24.5	32.8
	20.9	22.2	21.4	21.7		
LEADTIME	26.0	30.9	28.2	28.7		
	22.7	26.9				
C (FIX)/C (IN)	24.2	26.2				
	10.1	11.3				
LEADTIME	10.9	10.9				
	12.6	15.6				
C (FIX)/C (IN)	13.3	15.3				
	2	32				
LEADTIME	2	64				
	4					
C (FIX)/C (IN)	4					
	4					
LEADTIME	4					
	4					
C (FIX)/C (IN)	4					
	4					
LEADTIME	4					
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C (FIX)/C (IN)	4					
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LEADTIME	4					
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C (FIX)/C (IN)	4					
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LEADTIME	4					
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C (FIX)/C (IN)	4					
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C (FIX)/C (IN)	4					
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LEADTIME	4					
	4					
C (FIX)/C (IN)	4					
	4					
LEADTIME	4					
	4					
C (FIX)/C (IN)	4					
	4					
LEADTIME	4					

Table X5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=		202.1	SOURCES OF AGGREGATE REPLENISHMENT COST			
		C (OUT)/C (IN)	C (FIX)/C (IN)	PERCENT OF TOTAL COST= 24.7		MEAN
MEAN	4	99	32	64	2	4
	8	16				
LEADTIME	2	21.1	20.3	16.6	24.8	20.8
	4	25.6	24.9	20.3	30.2	20.6
C (FIX)/C (IN)	32	25.2	24.2	23.1	35.5	29.8
	64	29.7	28.8	29.8	28.8	
LEADTIME	2	50.9	49.1	39.8	60.2	50.5
	4					
C (FIX)/C (IN)	32					
	64					
LEADTIME	2	10.3	10.0			
	4	15.4	14.9			
C (FIX)/C (IN)	32	9.9	9.5			
	64	15.3	14.7			

Table X6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.0947

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN	
	4	99	32	64	2	4	8	16
	0.1781	0.0114	0.0948	0.0947	0.0936	0.0959	0.0950 0.0945	
9	0.1790	0.0110	0.0923	0.0978	0.0939	0.0962		
16	0.1772	0.0118	0.0973	0.0916	0.0933	0.0957		

## LEADTIME

2	0.1765	0.0106	0.0935	0.0936
4	0.1797	0.0122	0.0960	0.0958

## C (FIX) / C (IN)

32	0.1780	0.0116
64	0.1782	0.0112

## LEADTIME C (FIX) / C (IN)

2	0.1762	0.0109
2	0.1769	0.0103
4	0.1798	0.0123
4	0.1795	0.0121

Table X7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0126

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
8	0.1516	0.0069	0.0125	0.0126	0.0108	0.0143	0.0161
16	0.1964	0.0088	0.0153	0.0169	0.0140	0.0181	0.0108
LEADTIME	0.1292	0.0060	0.0111	0.0105	0.0092	0.0124	
2	0.1353	0.0058	0.0107	0.0109			
4	0.1679	0.0081	0.0142	0.0144			
C (FIX) / C (IN)	0.1468	0.0070					
32	0.1563	0.0068					
64							
LEADTIME C (FIX) / C (IN)	0.1302	0.0059					
2	0.1403	0.0057					
4	0.1635	0.0081					
64	0.1724	0.0080					



Table X8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2759

MEAN	C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
	0.2806	0.2711	0.3139	0.2379	0.2798	0.2720	0.2292 0.3226
8	0.2339	0.2245	0.2627	0.1957	0.2302	0.2282	
16	0.3274	0.3178	0.3651	0.2800	0.3293	0.3158	
LEADTIME							
2	0.2837	0.2758	0.3208	0.2387			
4	0.2775	0.2665	0.3070	0.2370			
C (FIX)/C (IN)							
32	0.3191	0.3087					
64	0.2422	0.2335					
LEADTIME C (FIX)/C (IN)							
2	0.3250	0.3167					
2	0.2425	0.2349					
4	0.3131	0.3008					
4	0.2419	0.2322					



Table XI6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCE OF AGGREGATE PERIOD-END INVENTORY  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		13.4															
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN									
		4	99	32	64	2	4	8	16								
		9.6	15.0	13.7	13.1	12.7	14.0	15.0	12.3								
		9.8	17.1	16.8	13.3	14.5	15.3										
		9.5	13.5	11.5	12.9	11.3	13.1										
		8.1	14.7	12.4	12.9												
		10.9	15.3	14.8	13.3												
		11.0	14.9														
		8.5	15.2														
		8.9	13.9														
		7.5	15.4														
		12.8	15.7														
		9.4	15.0														

LEADTIME C (FIX)/C (IN)

2 32  
2 64  
4 32  
4 64

C (FIX)/C (IN)

32  
64

LEADTIME

2  
4

MEAN

8  
16

Table X17

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			SOURCES OF AGGREGATE BACKLOG COST (% EXCESS OVER DP)			MEAN
		19.2				
		C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		
	4	99	32 64	2 4	8 16	
		-1.6 46.4	21.5 17.0	13.1 24.2	18.2 19.9	
MEAN						
	8	0.3 40.9	14.1 22.1	12.4 23.0		
	16	-2.9 50.8	27.2 13.1	13.6 25.1		
LEADTIME						
	2	-2.8 33.7	14.9 11.4			
	4	-0.6 57.1	27.1 21.5			
C (FIX)/C (IN)						
	32	-0.5 49.4				
	64	-2.6 43.5				
LEADTIME C (FIX)/C (IN)						
	2	-2.4 36.4				
	4	-3.1 31.1				
	8	1.1 60.5				
	16	-2.2 53.8				



Table X18

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE REPLENISHMENT COST  
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM=		0.3															
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN									
		4	99	32	64	2	4	8	16								
		6.8	-5.7	-0.1	0.6	-0.8	1.5	0.8	-0.0								
		7.8	-5.5	0.8	0.8	-0.9	2.6										
		6.2	-5.8	-0.8	0.4	-0.7	0.6										
		4.4	-5.6	0.0	-1.3												
		9.5	-5.7	-0.3	2.6												
		4.9	-4.9														
		8.2	-6.2														
		3.8	-3.6														
		4.8	-6.9														
		6.1	-6.2														
		11.8	-5.4														

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

64

32

64



Table X20

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 STATISTICAL INFORMATION FROM 24-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=			WEIGHTED PROPORTION OF DEMAND BACKLOGGED (% EXCESS OVER DP)		
19.2					
LEADTIME	C (FIX) / C (IN)	C (OUT) / C (IN)	C (FIX) / C (IN)		LEADTIME
			32	64	
2	4	99	32	64	2
4					4
8					8
16					16
MEAN					MEAN
2					2
4					4
8					8
16					16
MEAN					MEAN
2					2
4					4
8					8
16					16
MEAN					MEAN
2					2
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8					8
16					16
MEAN					MEAN
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16					16
MEAN					MEAN
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16					16
MEAN					MEAN
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4					4
8					8
16					16
MEAN					MEAN
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16					16
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MEAN					MEAN
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16					16
MEAN					MEAN
2					2
4					4
8					8
16					16
MEAN					MEAN
2					2
4					4
8					8
16					16
MEAN					MEAN
2					2
4					





## Appendix Y

### Multi-item Forecasts for the (24,24) Statistical Power Approximation

#### Model IV

Forecasting Properties of Inventory System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information, Revision Taking Place Every 24 Periods Using a 24-Period Demand History and Regression Estimates of Demand Means and Variances. Forecasts Made at Each Revision Using a 24-Period Demand History.

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Note: The corresponding appendices in MacCormick (1974), are his appendices T and U.

Table Y1

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 739.3				SOURCES OF TOTAL COST			
MEAN	LEADTIME	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME	MEAN		
4	99	32	64	2	4	8	16
38.8	61.2	44.9	55.1	47.5	52.5	42.2	57.8
16.2	26.0	19.0	23.2	20.0	22.2		
22.6	35.2	25.9	31.9	27.6	30.3		
18.7	28.9	21.2	26.3				
20.1	32.4	23.7	28.7				
17.0	27.9						
21.8	33.3						
8.1	13.1						
10.5	15.8						
8.9	14.9						
11.3	17.5						

Table Y2

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=				SOURCES OF AGGREGATE PERIOD-END INVENTORY						
470.4				PERCENT OF TOTAL COST= 63.6						
C (OUT)/C (IN)				C (FIX)/C (IN)				LEADTIME		MEAN
4 99				32 64				2 4		8 16
28.3 71.7				46.9 53.1				46.3 53.7		42.6 57.4
11.8 30.8				20.2 22.4				19.8 22.8		
16.6 40.8				26.7 30.7				26.5 30.9		
13.3 33.0				21.4 24.6						
15.1 38.7				25.5 28.2						
13.0 34.0										
15.4 37.7										
6.0 15.5										
7.3 17.5										
7.0 18.5										
8.1 20.1										

Table Y3

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF AGGREGATE BACKLOG COST			
OVERALL AGGREGATE POE SYSTEM=	66.9	PERCENT OF TOTAL COST= 9.0	
	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME
4	99	32 64	2 4
76.0	24.0	46.4 53.6	47.7 52.3
MEAN			MEAN
8			8 16
16			41.5 58.5
LEADTIME			
2	32.2 9.3	18.0 23.5	18.9 22.6
4	43.8 14.7	28.4 30.1	28.8 29.7
2	35.9 11.8	22.6 25.1	
4	40.1 12.2	23.8 28.6	
C (FIX)/C (IN)			
32	35.4 11.0		
64	40.6 13.1		
LEADTIME C (FIX)/C (IN)			
2	17.0 5.6		
2	18.9 6.2		
4	18.4 5.3		
4	21.7 6.9		



Table Y4

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS MODEL IV					
FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES					
SOURCES OF AGGREGATE REPLENISHMENT COST					
OVERALL AGGREGATE FOR SYSTEM=	202.0	PERCENT OF TOTAL COST=		27.3	
	C (OUT)/C (IN)	C (FIX)/C (IN)	LEADTIME		MEAN
	4 99	32 64	2 4		8 16
	50.8 49.2	39.8 60.2	50.4 49.6		41.4 58.6
MEAN					
8	21.1 20.3	16.6 24.8	20.7 20.7		
16	29.7 28.9	23.2 35.4	29.7 28.9		
LEADTIME					
2	25.6 24.8	20.2 30.2			
4	25.2 24.3	19.6 30.0			
C (FIX)/C (IN)					
32	20.3 19.5				
64	30.5 29.7				
LEADTIME C (FIX)/C (IN)					
2	10.3 9.9				
2	15.3 14.9				
4	10.0 9.6				
4	15.3 14.8				

Table Y5

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM= 0.0892				BACKLOG FREQUENCY			
C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN	
4	59	32	64	2	4	8	16
0.1742 0.0042		0.0875 0.0909		0.0897 0.0886		0.0888 0.0895	
0.1740 0.0036		0.0838 0.0939		0.0892 0.0884			
0.1743 0.0047		0.0911 0.0879		0.0903 0.0887			
0.1754 0.0041		0.0891 0.0904					
0.1729 0.0043		0.0858 0.0914					
0.1710 0.0040							
0.1773 0.0044							
0.1743 0.0040							
0.1766 0.0042							
0.1677 0.0040							
0.1781 0.0046							

Table Y6

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0068

MEAN	C (OUT)/C (IN)	C (FIX)/C (IN)			LEADTIME			MEAN
	4	32	64		2	4		8
8	0.1323 0.0017	0.0063 0.0073			0.0064 0.0071			0.0084 0.0059
16	0.1681 0.0020	0.0073 0.0095			0.0077 0.0092			
	0.1144 0.0016	0.0058 0.0061			0.0058 0.0060			
LEADTIME	2	0.1250 0.0017	0.0061 0.0068					
	4	0.1396 0.0017	0.0064 0.0077					
C (FIX)/C (IN)	32	0.1233 0.0015						
	64	0.1412 0.0018						
LEADTIME	2	0.1183 0.0016						
	2	0.1316 0.0017						
	4	0.1283 0.0015						
	4	0.1509 0.0019						

Table Y7

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODPL IV  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTPM= 0.2758

MEAN	C (OUT) / C (IN)		C (FIX) / C (IN)		LEADTIME		MEAN
	4	99	32	64	2	4	
8	0.2806	0.2709	0.3140	0.2376	0.2788	0.2728	0.2288
16	0.2334	0.2242	0.2618	0.1958	0.2290	0.2286	0.3228
LPACTIME	0.3279	0.3177	0.3662	0.2793	0.3286	0.3169	
2	0.2832	0.2744	0.3193	0.2383			
4	0.2781	0.2674	0.3087	0.2368			
C (FIX) / C (IN)	0.3202	0.3078					
32	0.2410	0.2341					
64							
LEADTIME C (FIX) / C (IN)	0.3250	0.3135					
2	0.2413	0.2353					
4	0.3154	0.3020					
4	0.2407	0.2329					



Table Y8

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

SOURCES OF VARIANCE OF TOTAL COST				
OVERALL AGGREGATE FOR SYSTEM= 1239.2				
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN
4	99	32 64	2 4	8 16
15.9	84.1	42.3 57.7	44.9 55.1	37.9 62.1
MEAN				
8	6.7 31.2	16.2 21.7	14.4 23.6	
16	9.2 52.9	26.1 36.0	30.6 31.5	
LEADTIME				
2	5.5 39.5	17.3 27.7		
4	10.4 44.6	25.1 30.0		
C(FIX)/C(IN)				
32	7.8 34.6			
64	8.1 49.5			
LEADTIME C(FIX)/C(IN)				
2	2.5 14.8			
2	3.0 24.7			
4	5.3 19.8			
4	5.2 24.8			



Table Y10

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## SOURCES OF S.D. OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM=		24.4															
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME											
		4	99	32	64	2	4	8	16								
		8.8	22.8	17.2	17.3	14.7	19.5	16.1	18.3								
		5.9	15.0	11.2	11.5	9.9	12.7										
		6.5	17.2	13.1	12.9	10.9	14.8										
		5.1	13.8	10.4	10.4												
		7.1	18.1	13.8	13.8												
		6.0	16.2														
		6.4	16.1														
		3.4	9.8														
		3.8	9.7														
		4.9	12.9														
		5.1	12.8														

MEAN

8

16

LEADTIME

2

4

C(FIX)/C(IN)

32

64

LEADTIME C(FIX)/C(IN)

2

4

8

16

Table Y11

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV  
FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

## SOURCES OF S.D. OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM=		26.3															
		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		LEADTIME		C (FIX)/C (IN)		LEADTIME		C (FIX)/C (IN)		LEADTIME	
		4 99		32 64		2 4		2 4		32 64		2 4		32 64		2 4	
		10.4 24.2		16.4 20.6		18.5 18.8		18.5 18.8		16.4 20.6		18.5 18.8		16.4 20.6		18.5 18.8	
		6.4 13.7		9.3 11.9		9.1 12.1		9.1 12.1		9.3 11.9		9.1 12.1		9.3 11.9		9.1 12.1	
		8.2 20.0		13.5 16.8		16.0 14.4		16.0 14.4		13.5 16.8		16.0 14.4		13.5 16.8		16.0 14.4	
		5.7 17.6		11.1 14.8		11.1 14.8		11.1 14.8		11.1 14.8		11.1 14.8		11.1 14.8		11.1 14.8	
		8.7 16.7		12.1 14.4		12.1 14.4		12.1 14.4		12.1 14.4		12.1 14.4		12.1 14.4		12.1 14.4	
		7.3 14.6															
		7.4 19.3															
		3.8 10.4															
		4.2 14.2															
		6.2 10.3															
		6.1 13.0															

MEAN

8

16

LEADTIME

2

4

C (FIX)/C (IN)

32

64

LEADTIME C (FIX)/C (IN)

2

32

2

64

4

32

4

64

MEAN

8 16

15.1 21.6



SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
MODEL IV

FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

OVERALL AGGREGATE FOR SYSTEM=				SOURCES OF S.D. OF AGGREGATE REPLENISHMENT COST				
5.9		C (OUT)/C (IN)		C (FIX)/C (IN)		LEADTIME		MEAN
4	99	32	64	2	4	8	16	
4.0	4.3	3.5	4.8	4.3	4.0	3.9	4.4	
MEAN								
8	2.7	2.9	2.1	3.4	2.7	2.8		
16	3.0	3.2	2.8	3.4	3.3	2.9		
LEADTIME								
2	2.9	3.2	2.7	3.4				
4	2.8	2.9	2.2	3.4				
C (PIX)/C (IN)								
32	2.3	2.6						
64	3.3	3.4						
LEADTIME C (FIX)/C (IN)								
2	1.8	2.0						
2	2.3	2.4						
4	1.6	1.6						
2	2.3	2.4						

Table Y13

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS  
 MODEL IV  
 FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS  
 USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES

S.D. OF BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM = 0.1844

MEAN	C (OUT) / C (IN)	C (FIX) / C (IN)	LEADTIME		MEAN
			2	4	
8	0.1803 0.0387	0.1296 0.1312	0.1212 0.1390		0.1357 0.1248
16	0.1331 0.0262	0.0935 0.0984	0.0890 0.1024		
	0.1216 0.0285	0.0898 0.0868	0.0822 0.0940		
LEADTIME					
2	0.1184 0.0256	0.0851 0.0863			
4	0.1359 0.0291	0.0978 0.0988			
C (FIX) / C (IN)					
32	0.1268 0.0267				
64	0.1281 0.0281				
LEADTIME C (FIX) / C (IN)					
2	0.0832 0.0176				
2	0.0842 0.0186				
4	0.0957 0.0201				
4	0.0965 0.0210				







Table Y16

SYSTEM OF 16 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS MODEL IV					FORECASTS, MADE EVERY 24 PERIODS, USING A 24-PERIOD HISTORY; CONTROL REVISION HISTORY 24 PERIODS USING REGRESSION ESTIMATES OF DEMAND MEANS AND VARIANCES				
OVERALL AGGREGATE FOR SYSTEM=					SOURCES OF AGGREGATE BACKLOG COST (% UNDERESTIMATE OF ACTUAL)				
46.1									
C (OUT)/C (IN)					C (FIX)/C (IN)				
4 99					32 64				
12.7 75.6					49.6 42.7				
					40.4 50.5				
					47.6 45.0				
MEAN					LEADTIME				
8					2 4				
16					8 16				
LEADTIME					MEAN				
2					45.3 49.5				
4					36.7 51.2				
C (FIX)/C (IN)									
32					43.0 37.8				
64					54.7 46.3				
LEADTIME C (FIX)/C (IN)									
2					9.1 73.2				
32					6.3 69.4				
64					21.5 81.6				
					12.5 75.7				









